COV886 Special Module in Algorithms: Computational Social Choice

Lecture 1

Voting Rules

Jan 15, 2022 | Rohit Vaish

Reminder about starting recording

What is Computational Social Choice?

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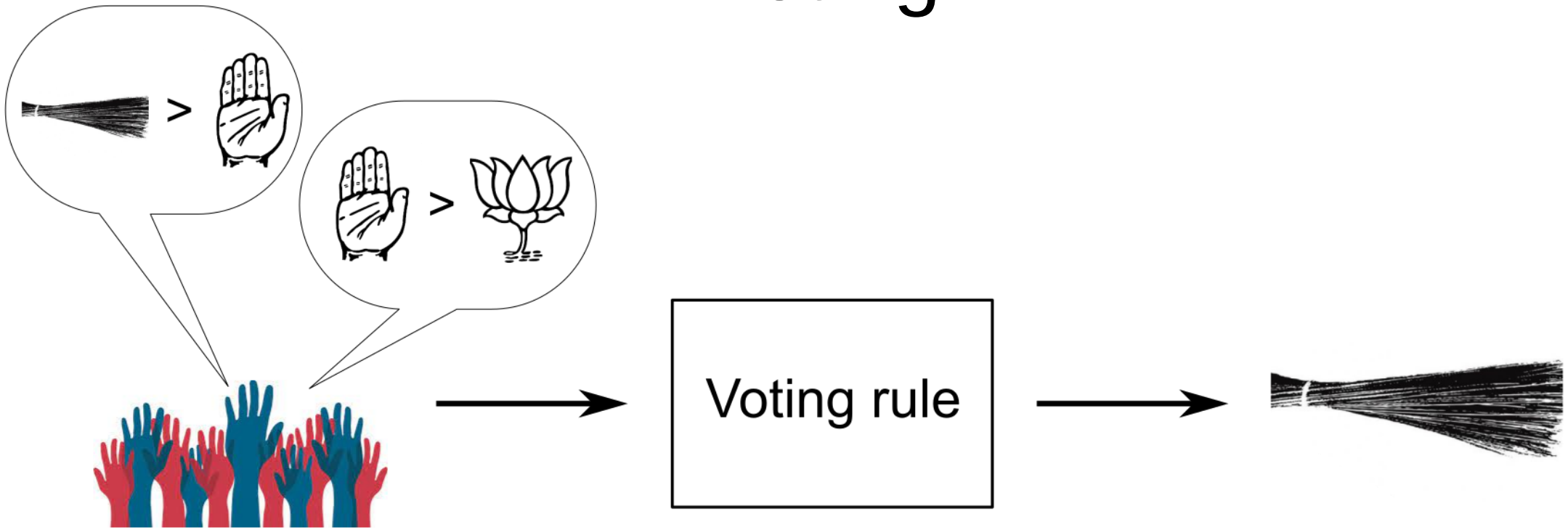
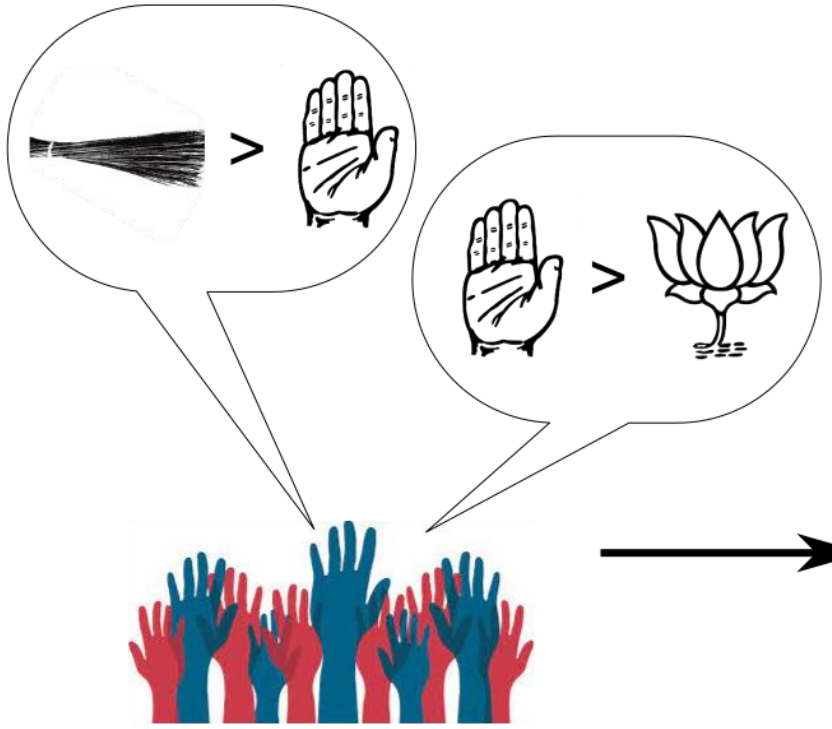
Understanding the role of computation
in collective decision-making problems

Voting



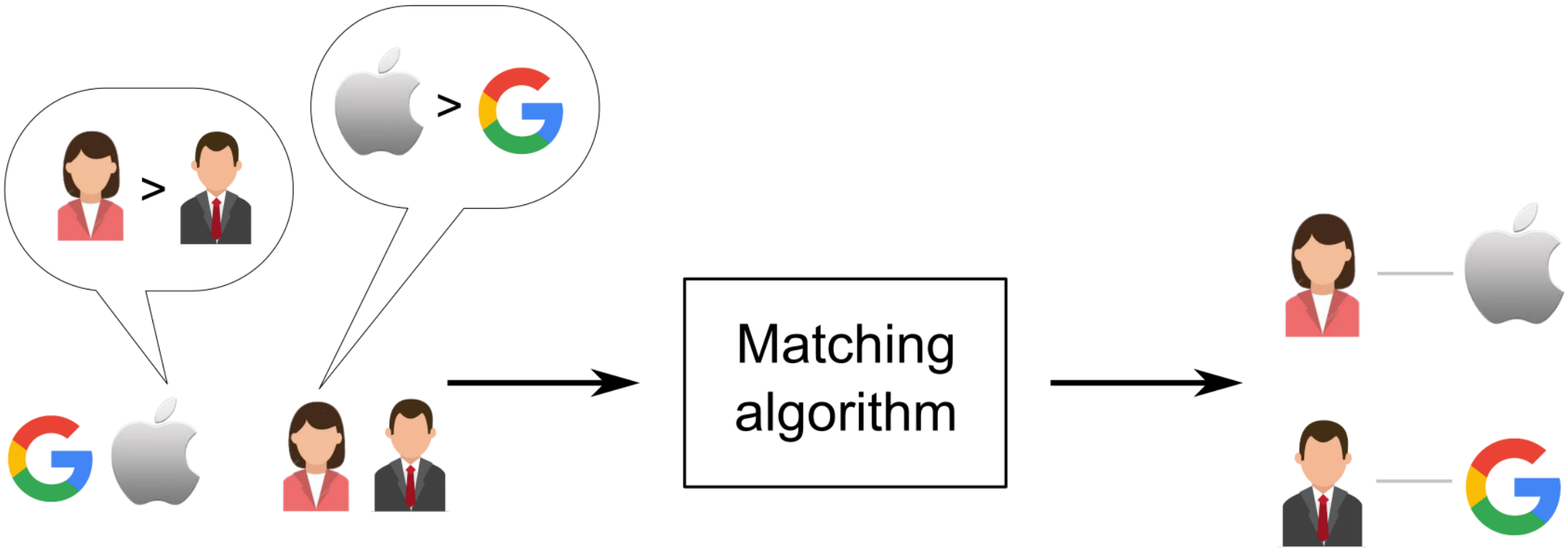
Voting



Voting rule

amazon

Matching



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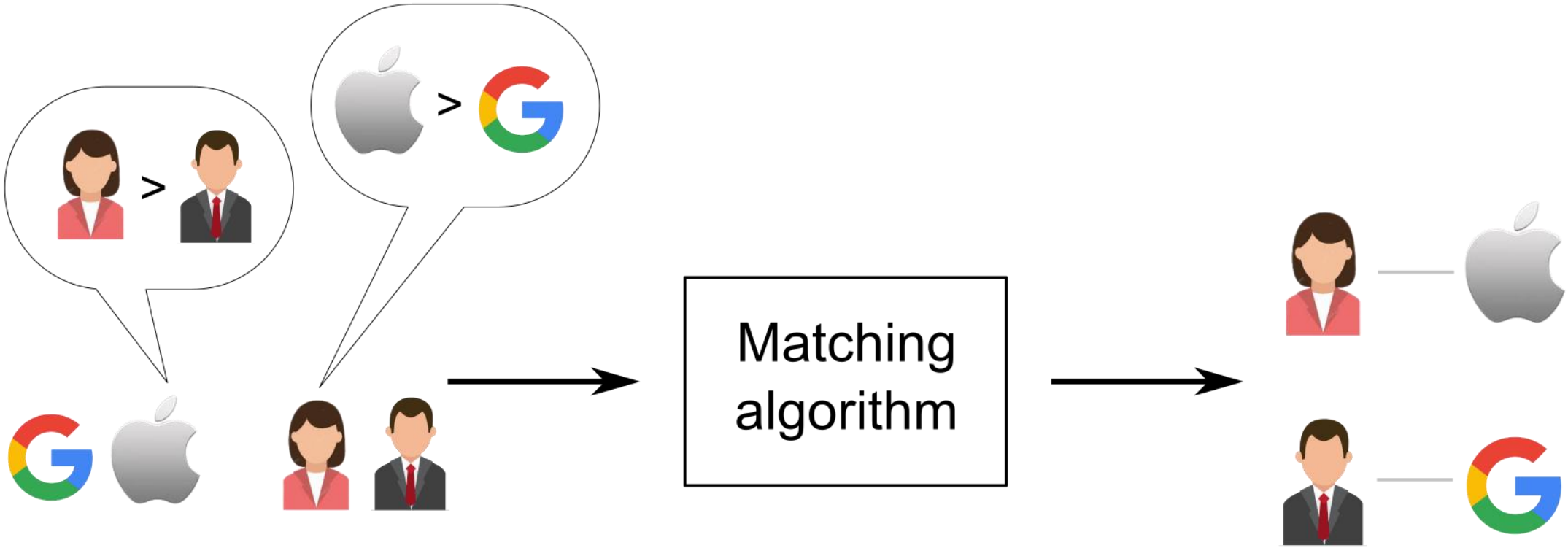
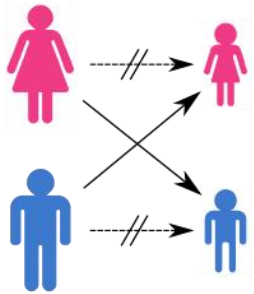
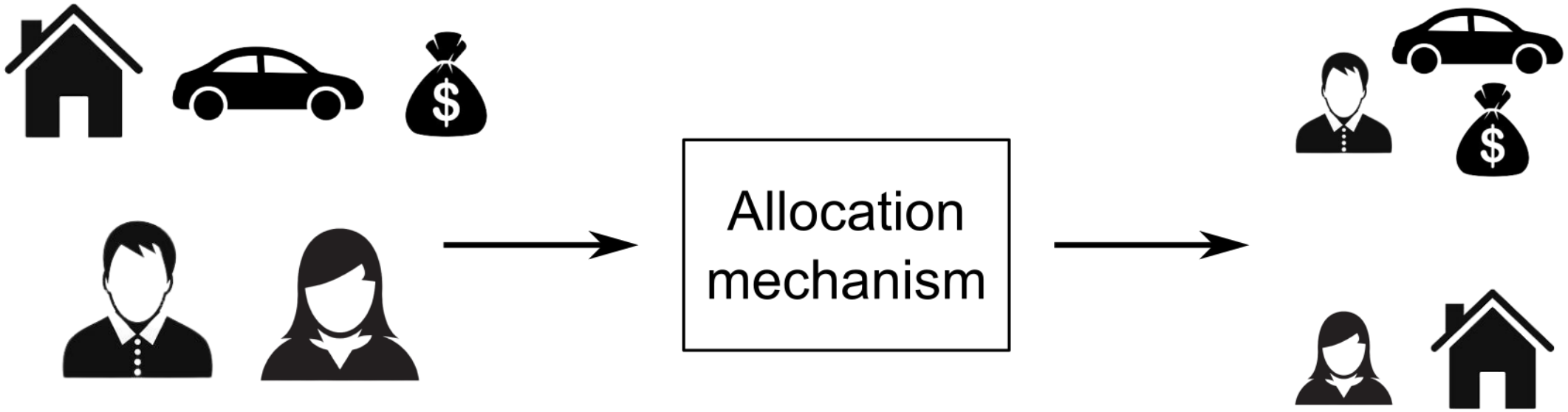




UBER

airbnb

Fair Division



Fair Division

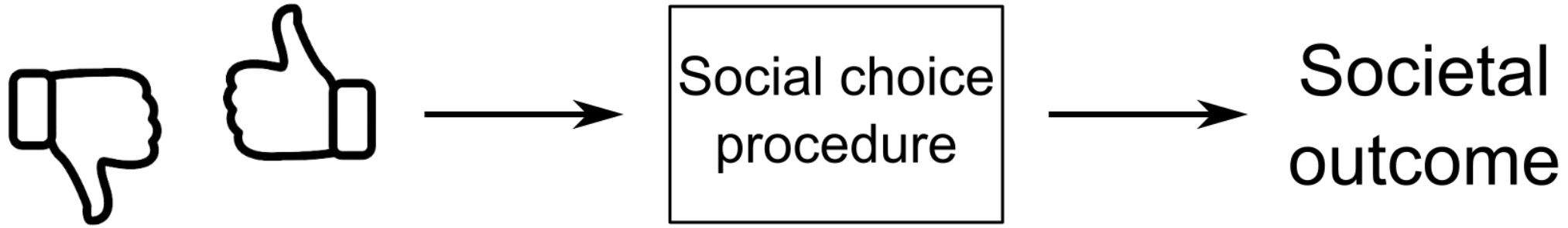
Allocation mechanism

Social Choice

Making a collective decision from individual preferences

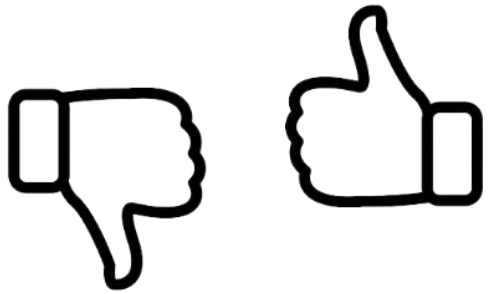
Social Choice

Making a collective decision from individual preferences



Social Choice

Making a collective decision from individual preferences

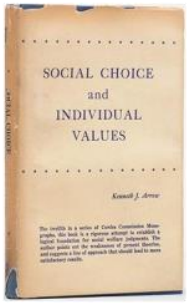


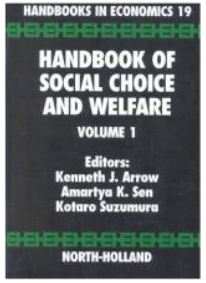
→

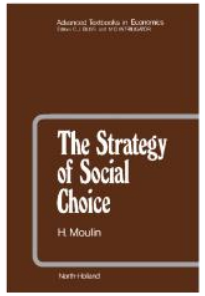
Social choice procedure

→

Societal outcome








Arrow


Sen


Maskin


Roth


Shapley

Classical Social Choice

Does there exist a social choice procedure
with the desired economic properties?

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Classical Social Choice

Does there exist a social choice procedure
with the desired economic properties?

Computational Social Choice

(This course)

How does computation influence
the economic properties of social choice procedures?


VOTE

[Lec 1]

Voting rules

[Lec 2]

Truthful elections are impossible

[Lec 3+4]

...but computation saves the day!

[Lec 1]

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[Lec 5]

Matching organ donors with patients

[Lec 6+7]

Stable matchings and incentives

[Lec 8+9]

Finding "fair" and "democratic" matchings

[Lec 1]

Voting rules

[Lec 2]

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[Lec 5]

Matching organ donors with patients

[Lec 6+7]

Stable matchings and incentives

[Lec 8+9]

Finding "fair" and "democratic" matchings

[Lec 10+11]

Cake-cutting and rent division

[Lec 12+13]

Dividing the indivisible

[Lec 14]

Fair allocation of seats in a parliament

Why is Voting Important

Mon 10 Jan	Tue 11 Jan	Wed 12 Jan	Thu 13 Jan	Fri 14 Jan	Sat 15 Jan
8:00 am 14	8:00 am 13	8:00 am 12	8:00 am 13	8:00 am 13	8:00 am 11
9:00 am 12	9:00 am 13	9:00 am 12	9:00 am 11	9:00 am 13	9:00 am 9
10:00 am 9	10:00 am 14	10:00 am 13	10:00 am 8	10:00 am 14	10:00 am 7
11:00 am 10	11:00 am 9	11:00 am 11	11:00 am 7	11:00 am 9	11:00 am 6
12:00 pm 10	12:00 pm 9	12:00 pm 9	12:00 pm 9	12:00 pm 9	12:00 pm 6
1:00 pm 11	1:00 pm 9	1:00 pm 11	1:00 pm 11	1:00 pm 9	1:00 pm 8
2:00 pm 9	2:00 pm 6	2:00 pm 5	2:00 pm 9	2:00 pm 7	2:00 pm
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4:00 pm 8	4:00 pm 5	4:00 pm 6	4:00 pm 7	4:00 pm 7	4:00 pm
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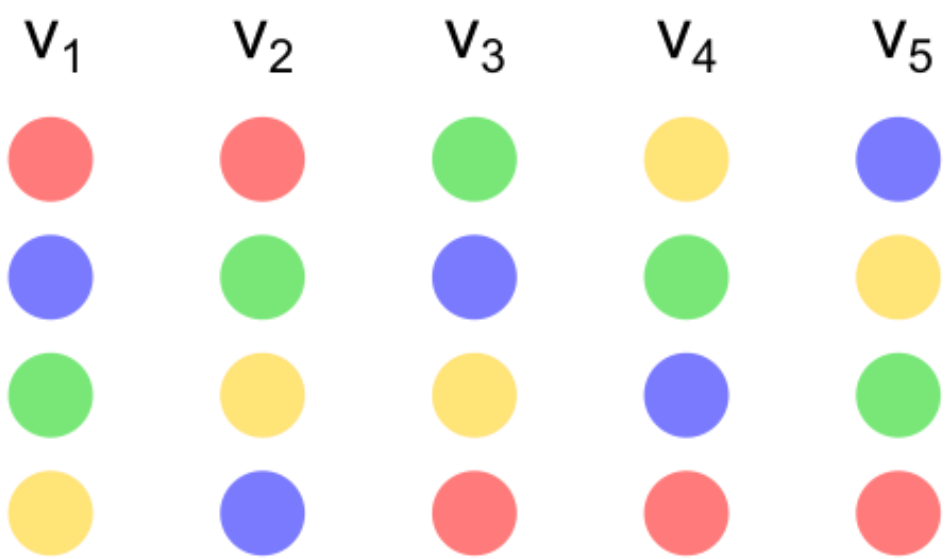
Voting Setup

Voting Setup



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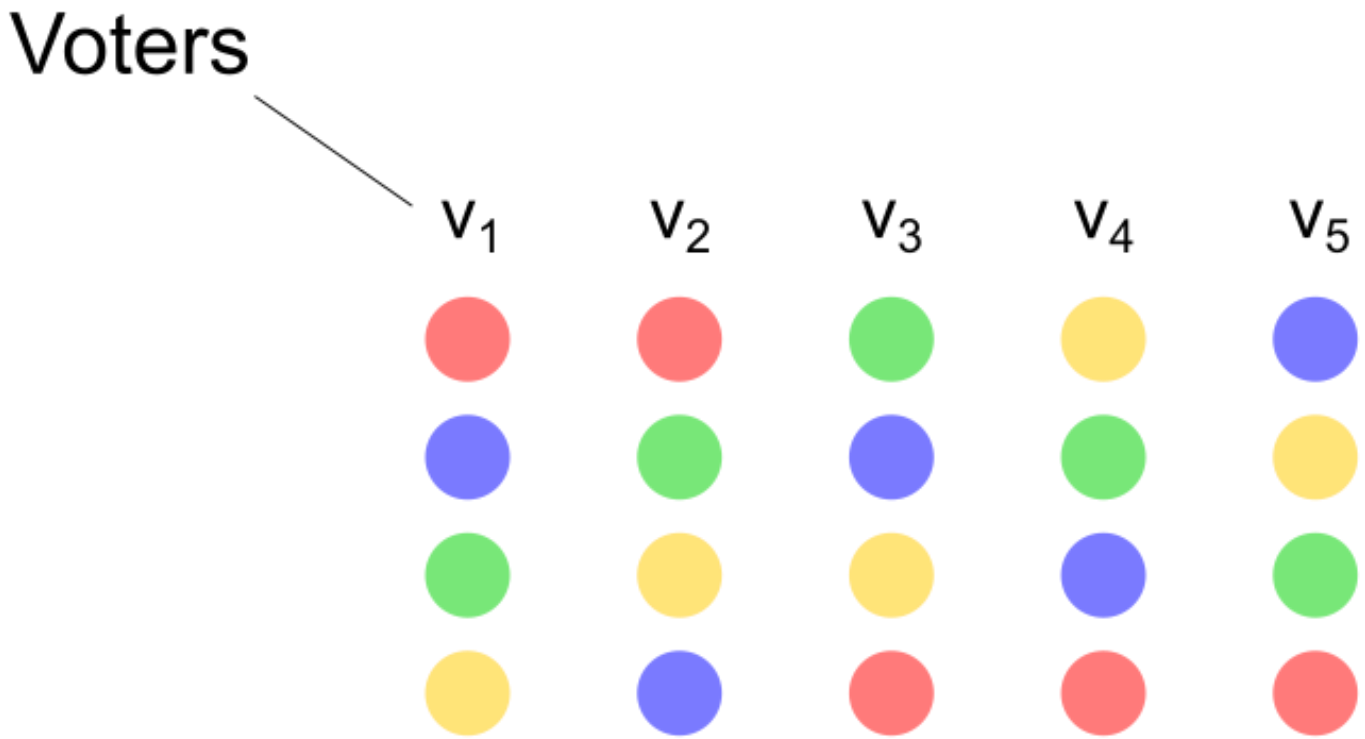

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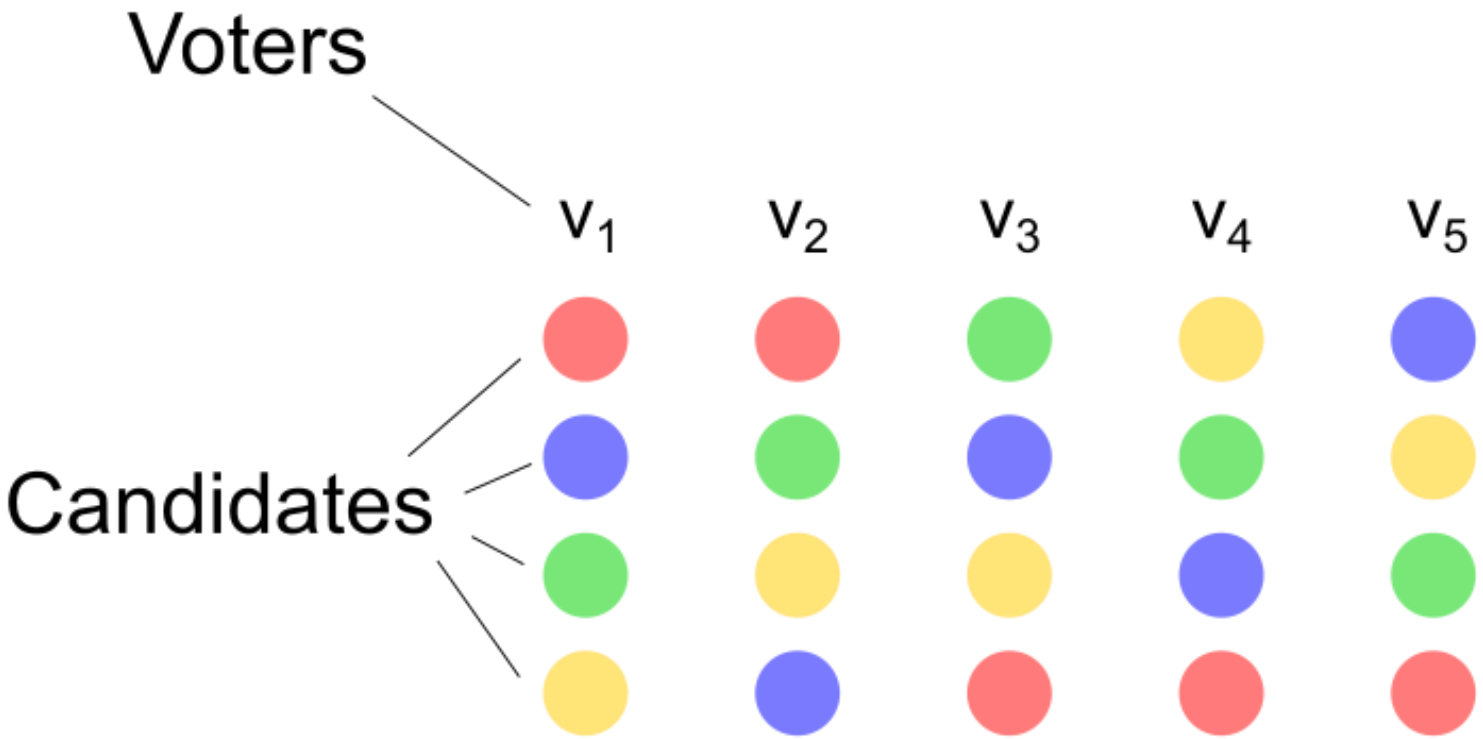

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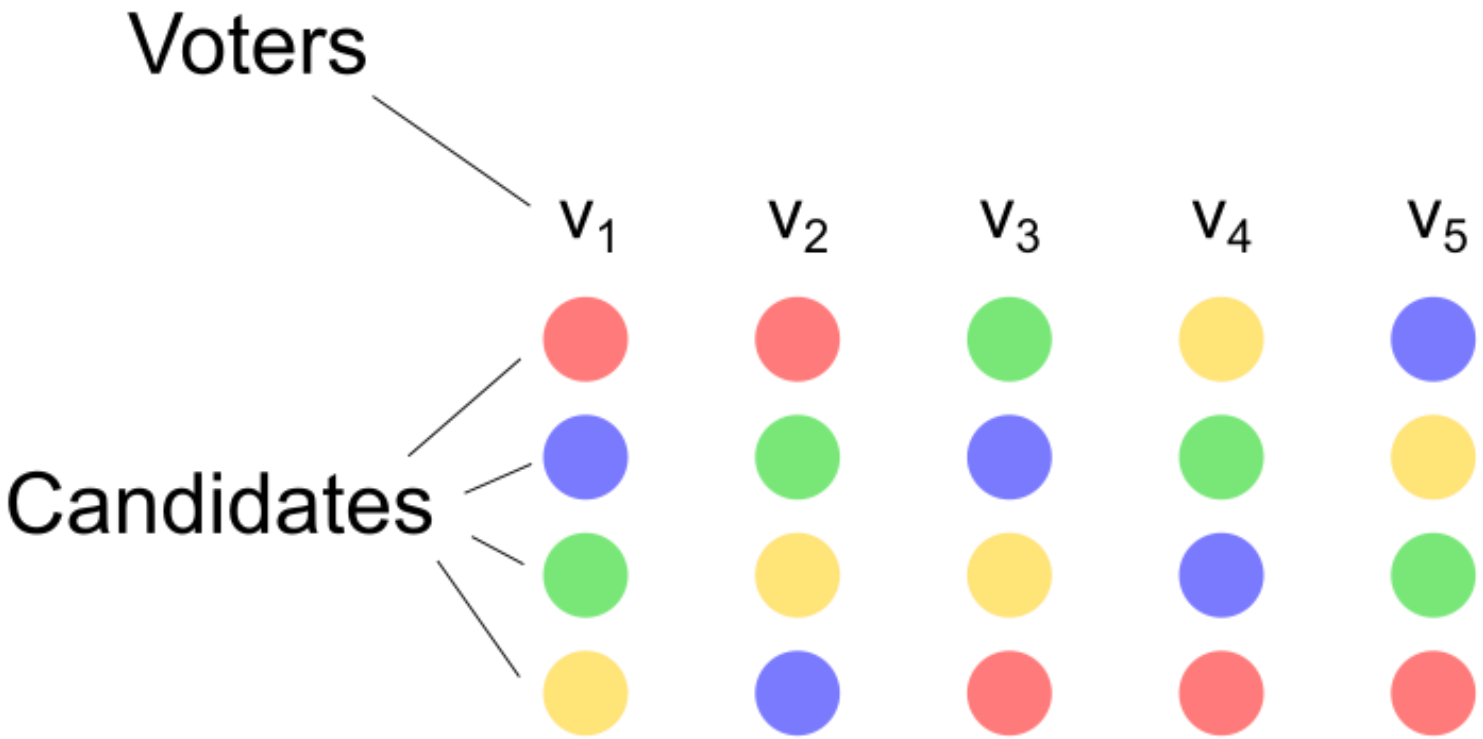

Goal: Pick exactly one winning candidate.

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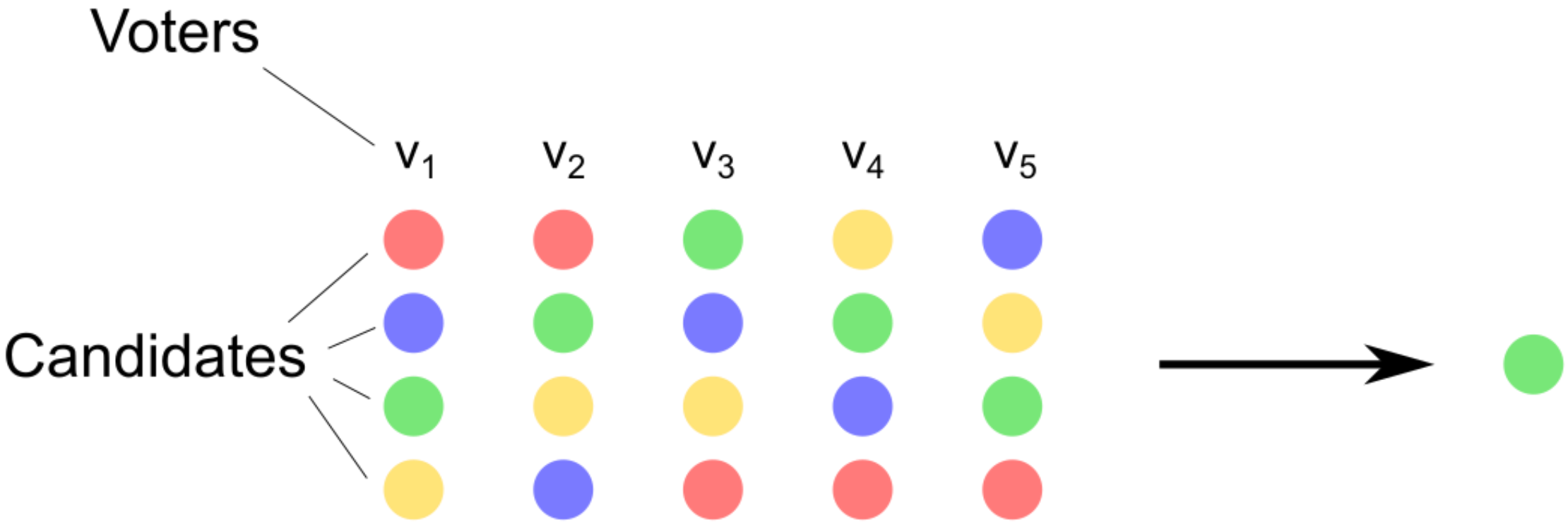

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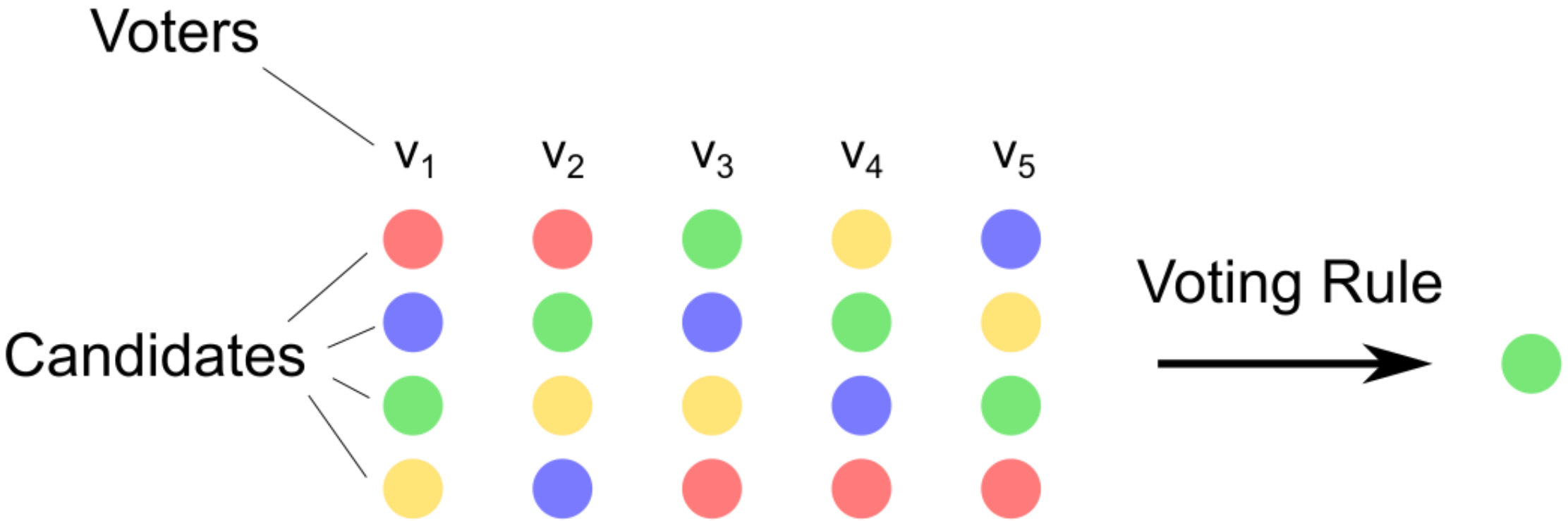

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Voting with Two Candidates

Voting with Two Candidates

Majority!

Voting with More Than Two Candidates

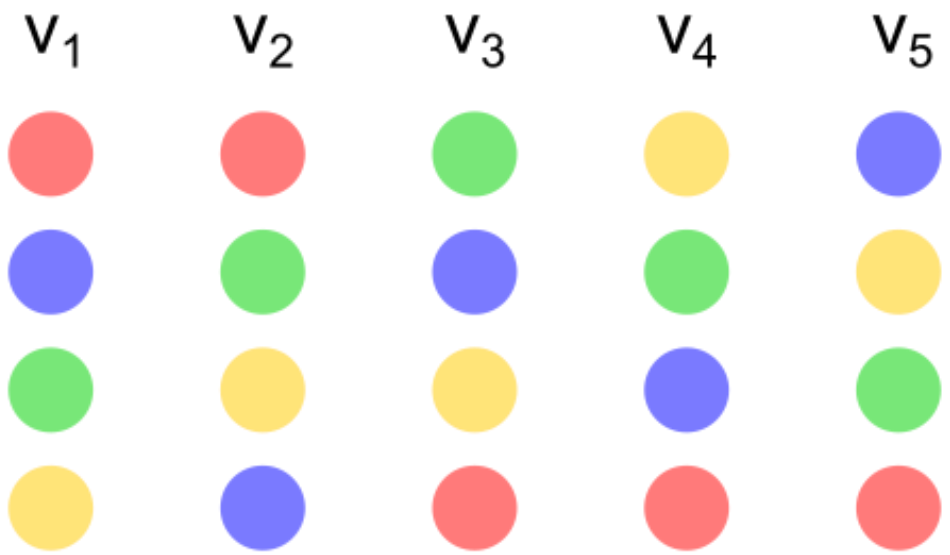
No candidate may have a majority…

1

Plurality

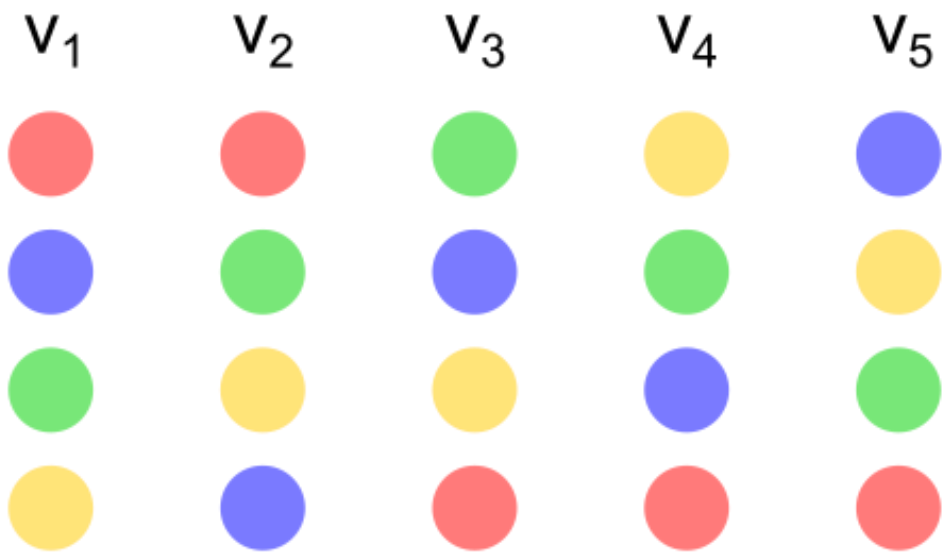
1

Plurality



1

Candidate with the most first-place votes wins



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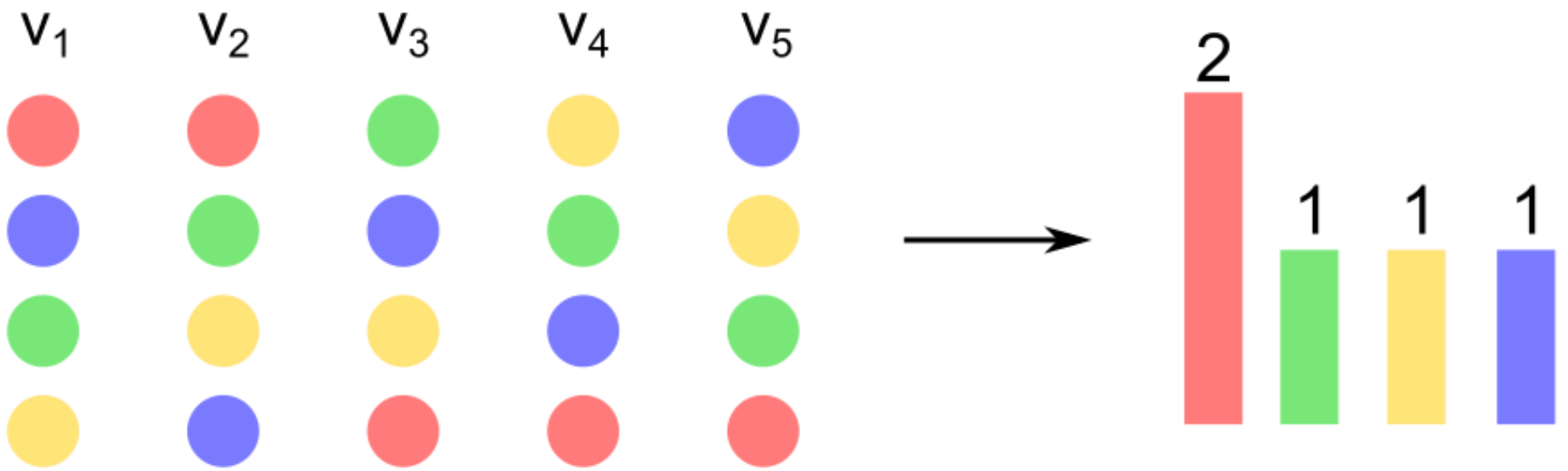

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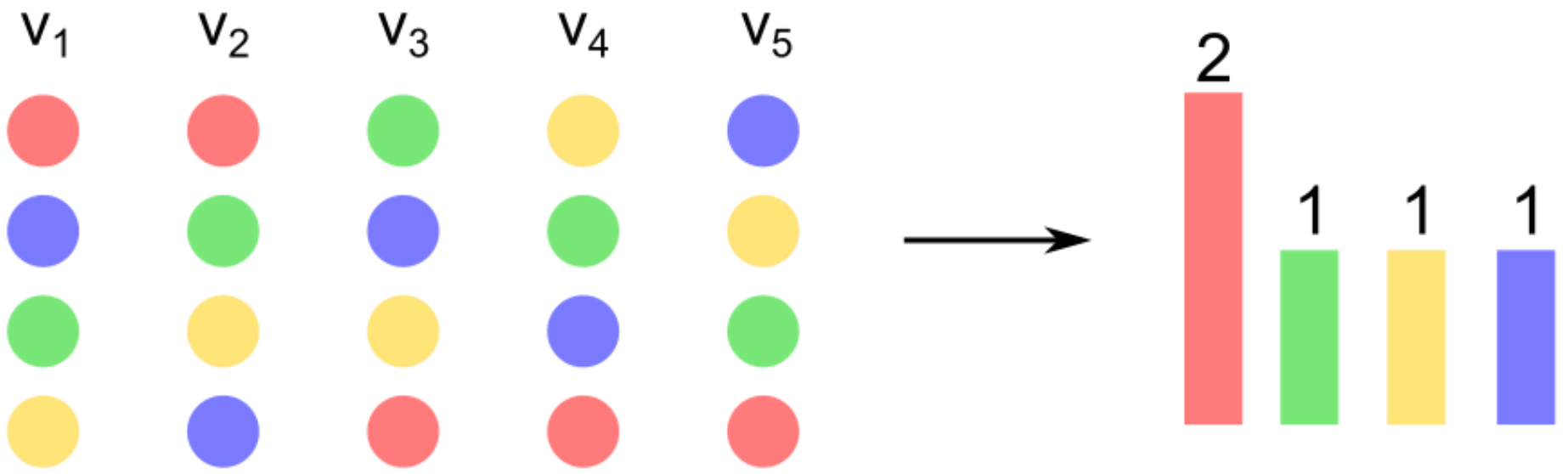


Plurality

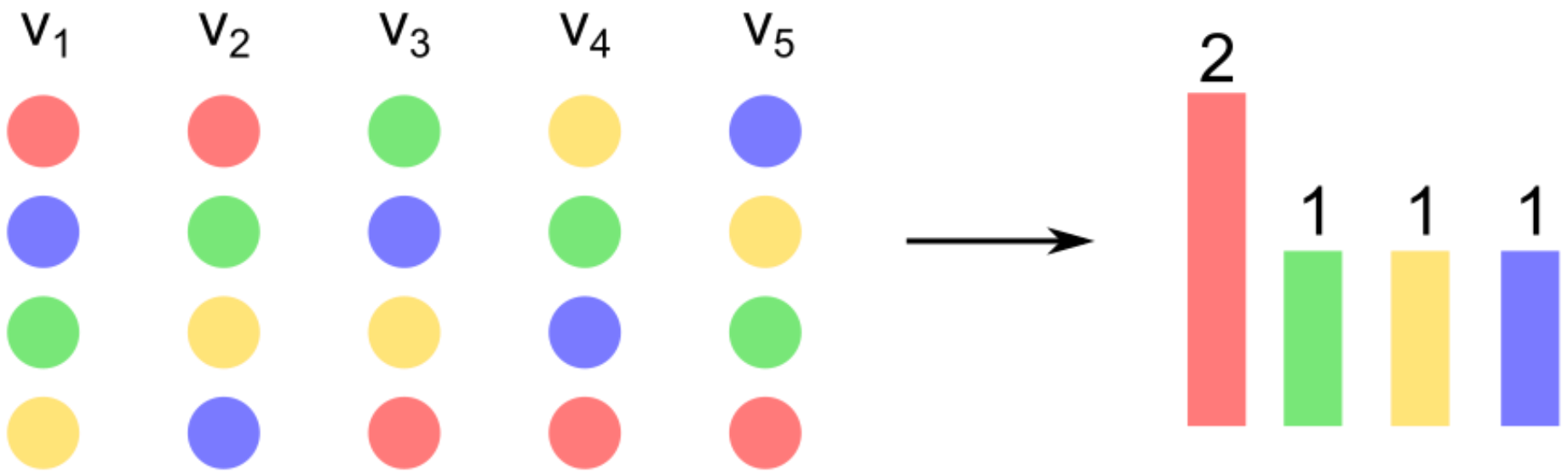
Plurality winner:

1

Plurality

Candidate with the most first-place votes wins*

*subject to tie-breaking: lexicographic, random, ...



Plurality winner:

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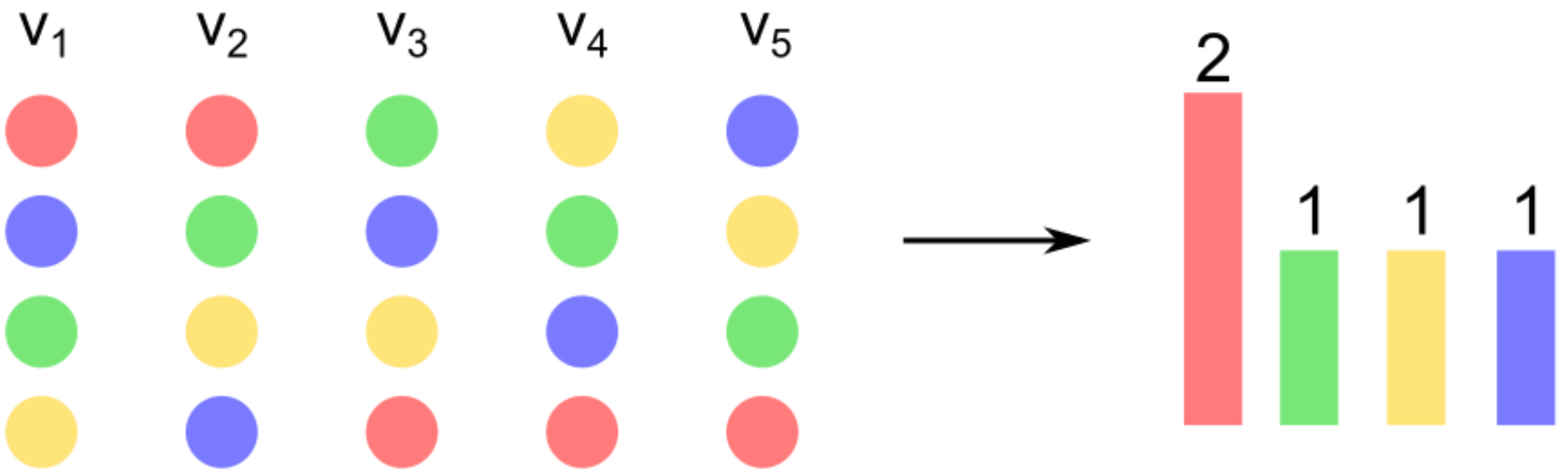
Problem: A majority prefers ● over the Plurality winner.

Plurality



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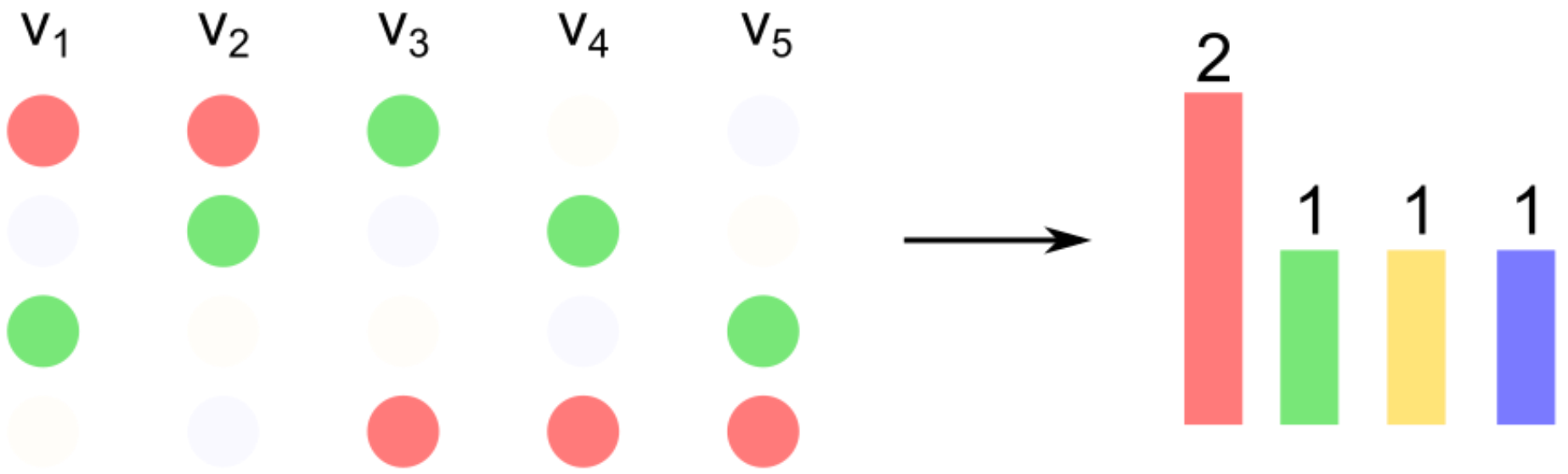

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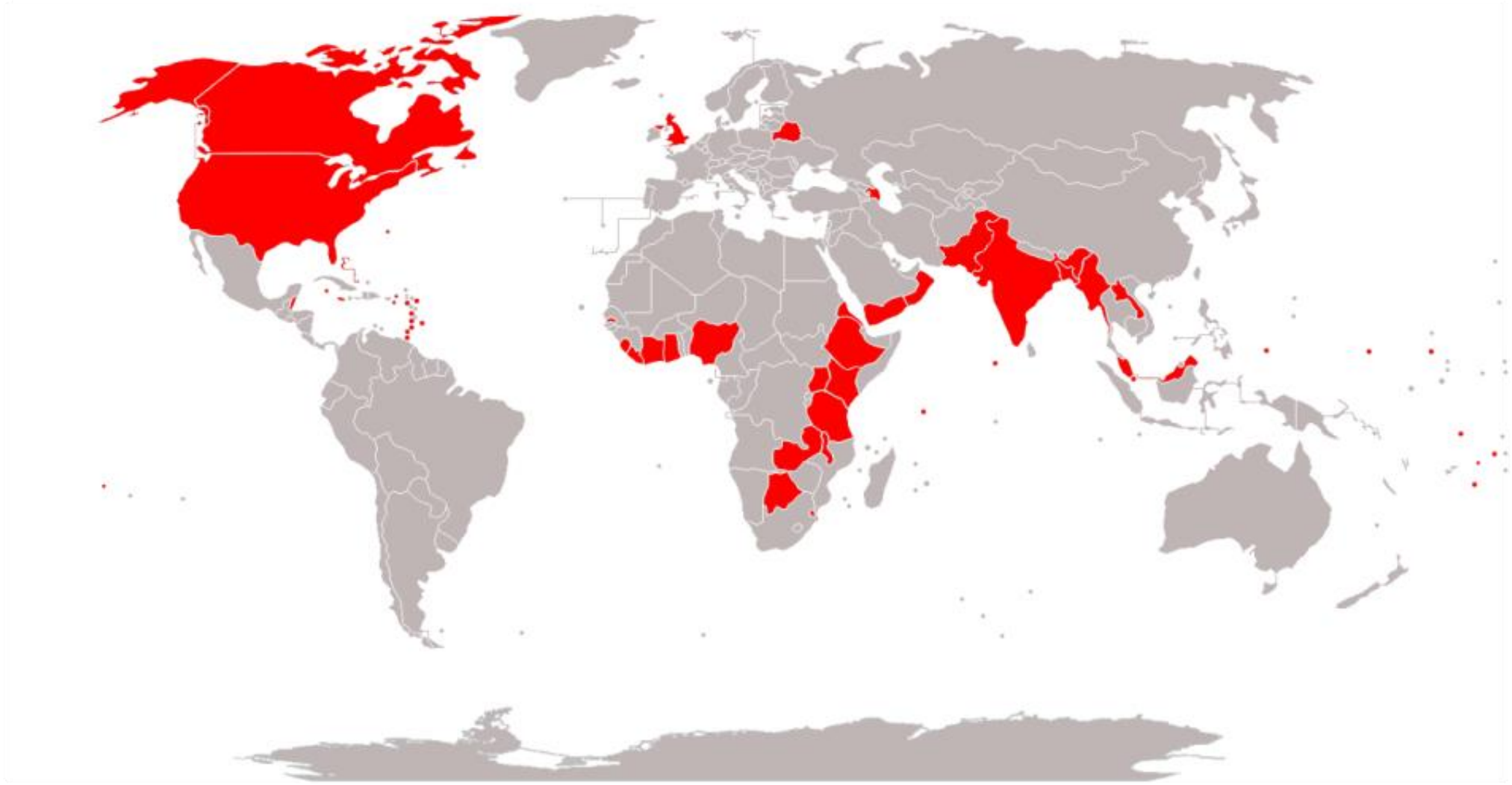

Image Source: Wikipedia article on "Electoral system" (Jan 2022)

2

Borda Count

2

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Jean-Charles de Borda
(1733-1799)

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Each voter gives its k^{th} ranked candidate m-k points, where m is the number of candidates.

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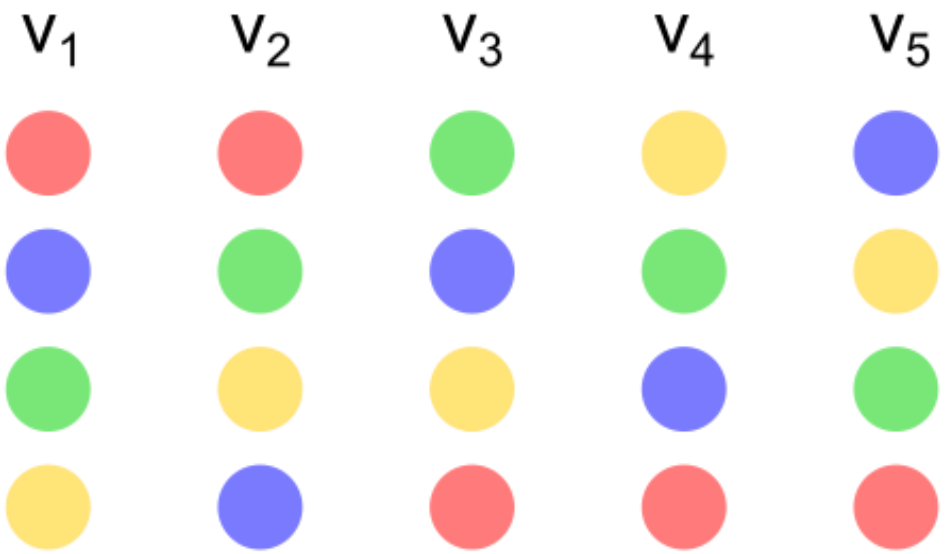

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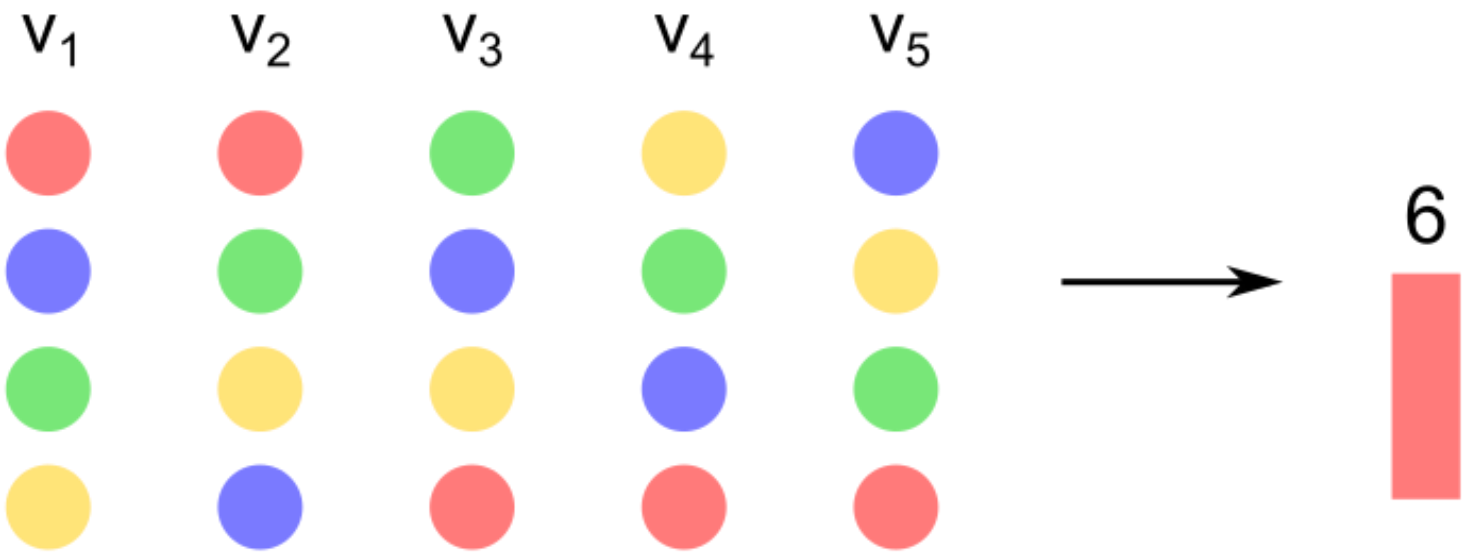

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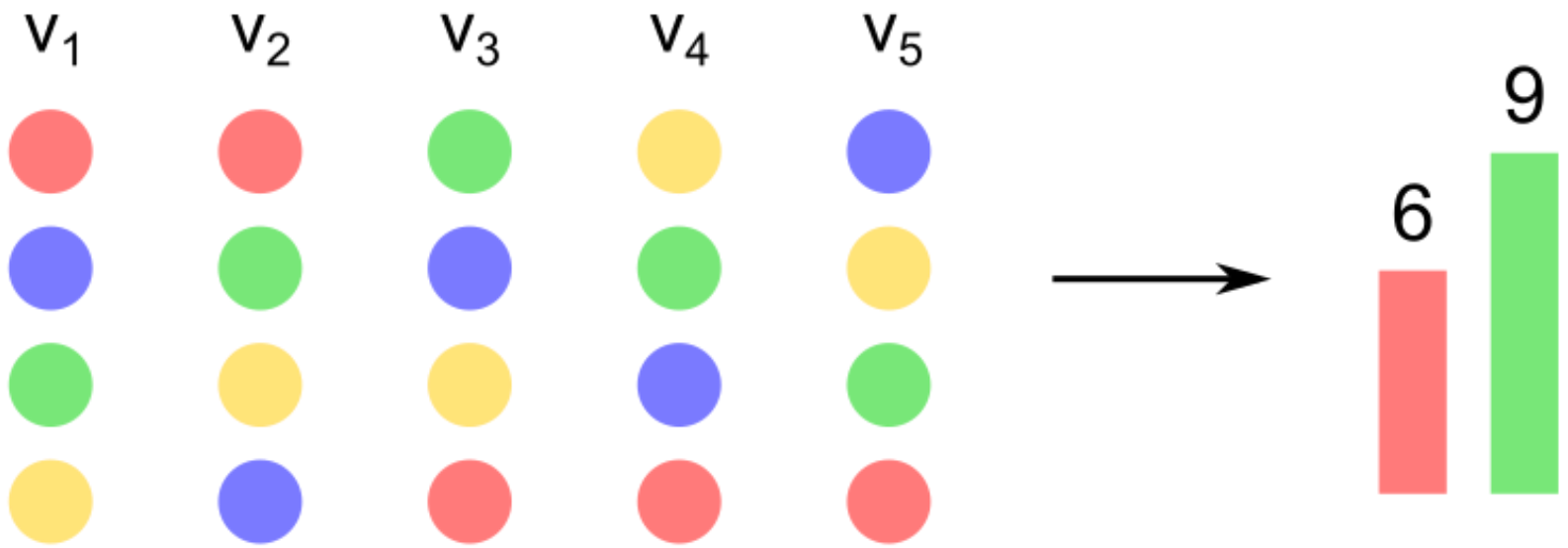

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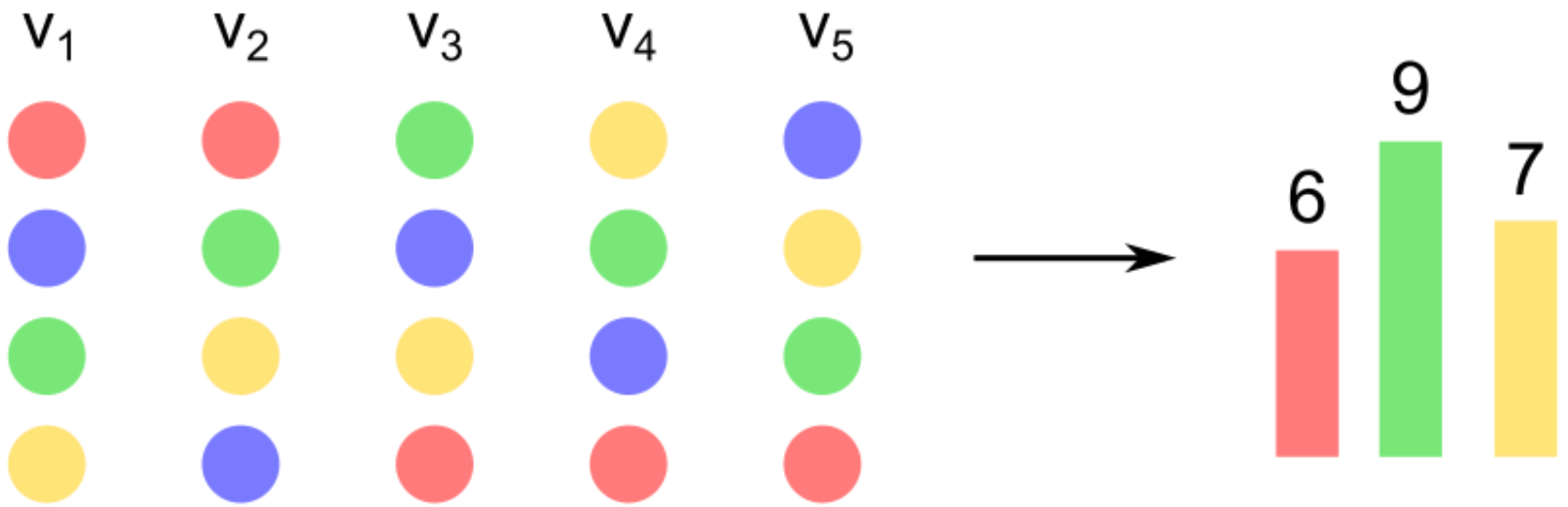

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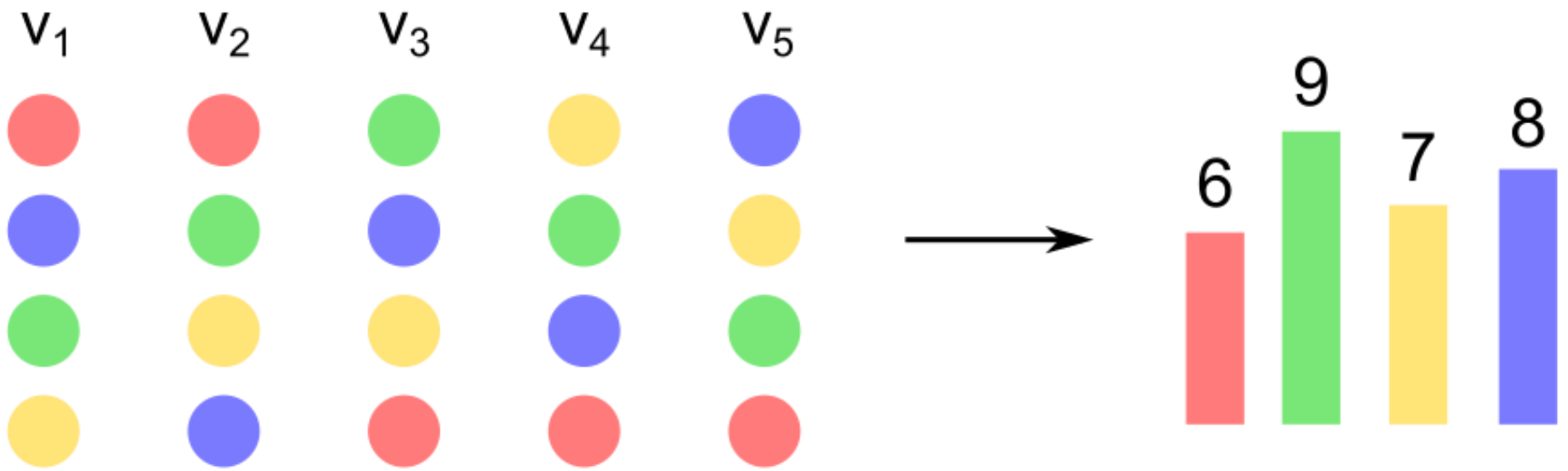

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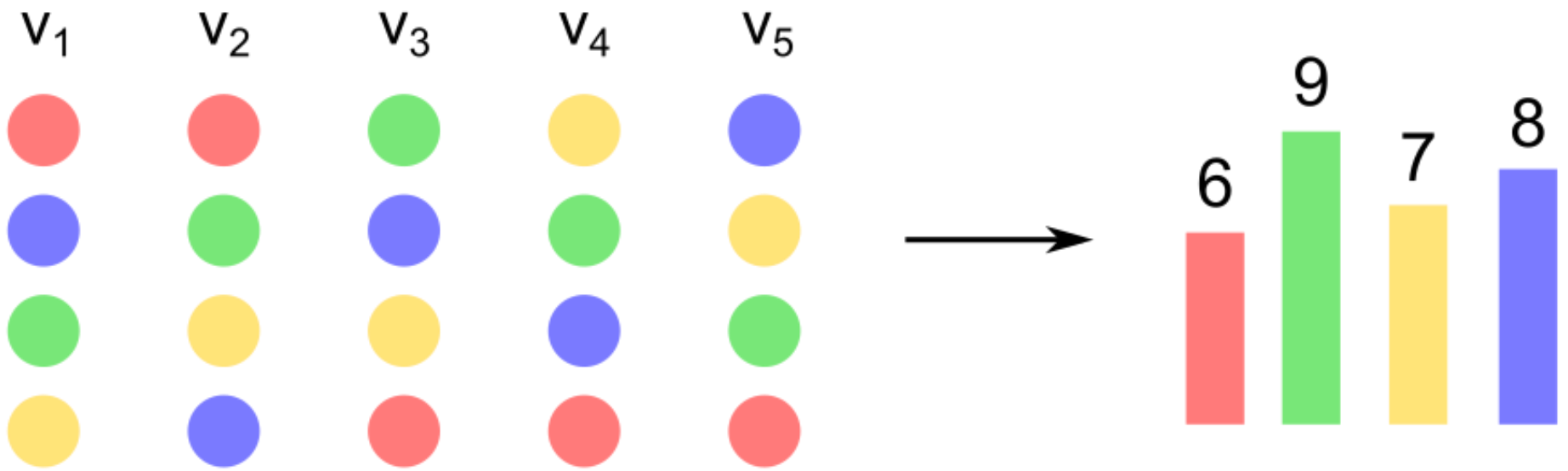

Borda winner:

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Problem: Susceptible to strategic voting (manipulation).

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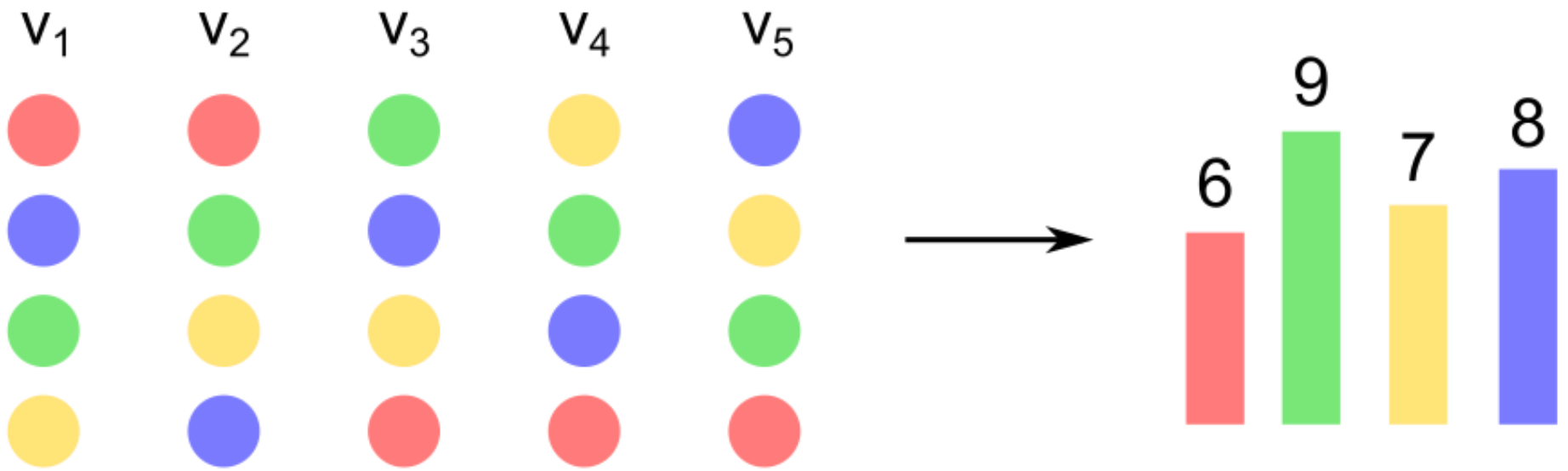

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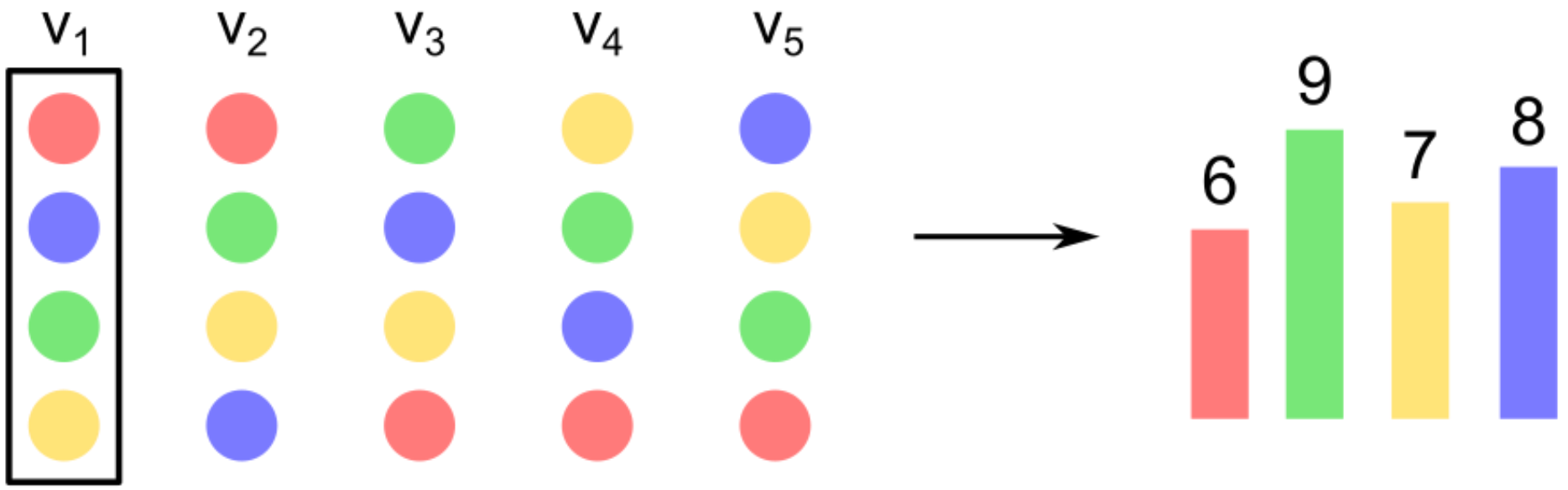

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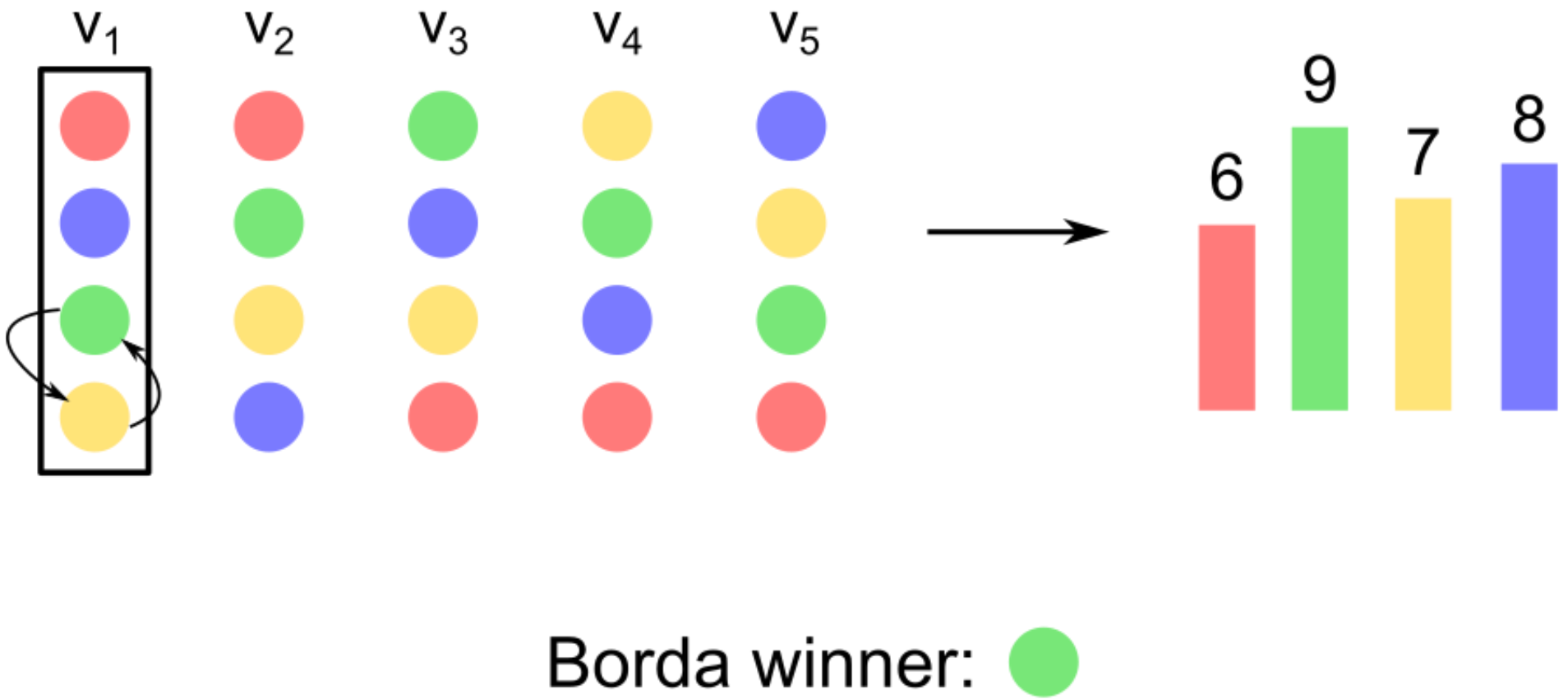

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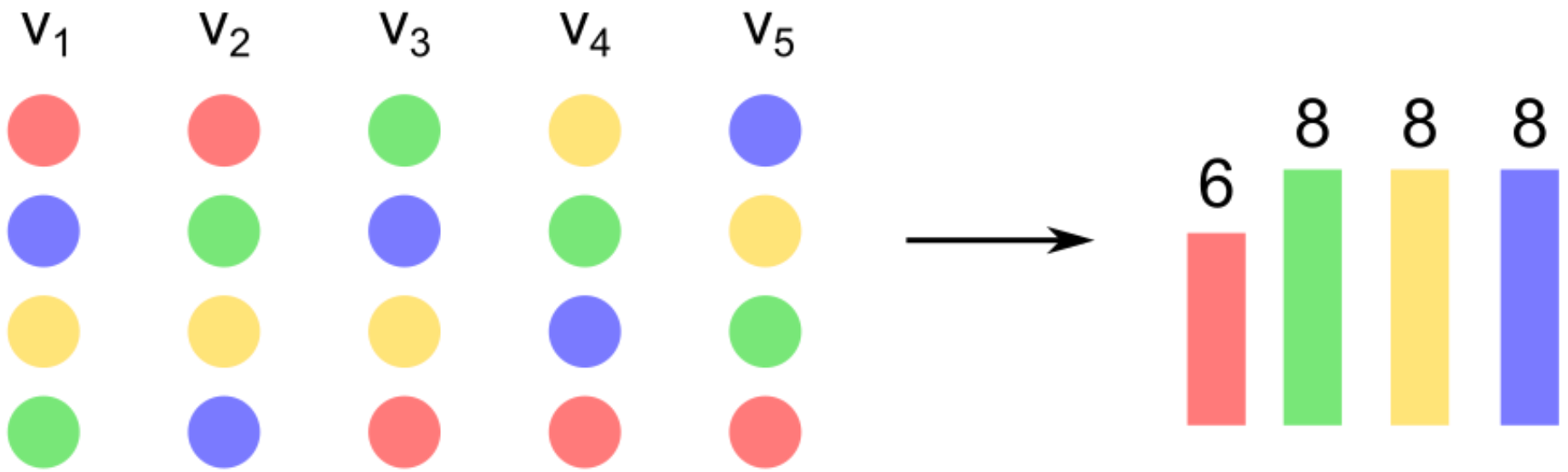

Borda winner:

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Borda Count

Problem: Susceptible to strategic voting (manipulation).

"My scheme is intended for only honest men."

2

Borda Count

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Plurality With Runoff

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The two candidates with the highest Plurality scores in the first round go head-to-head in the next round

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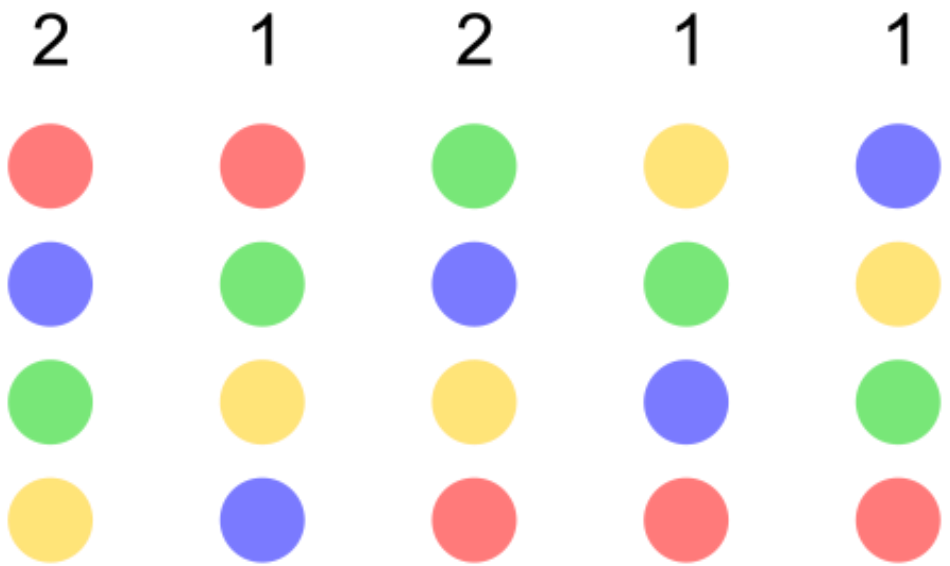

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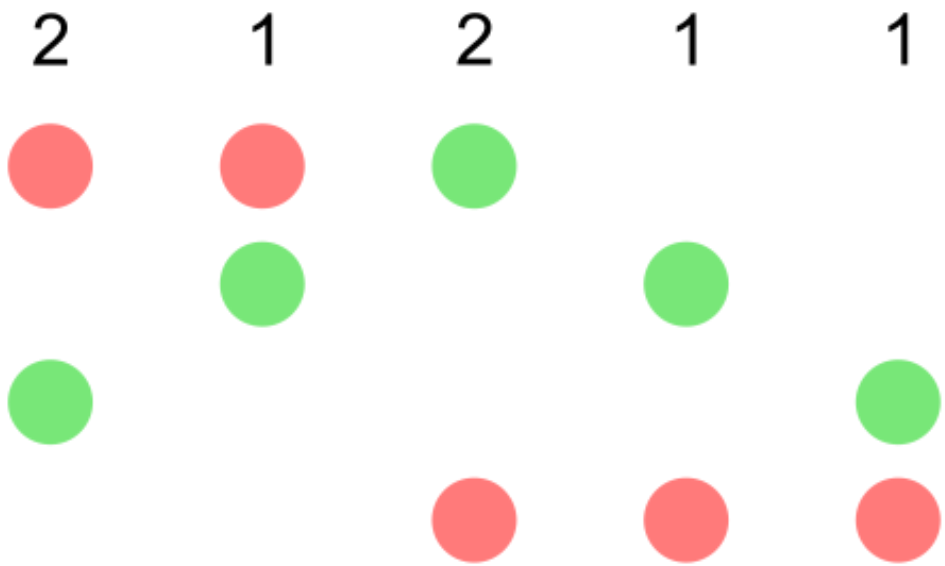

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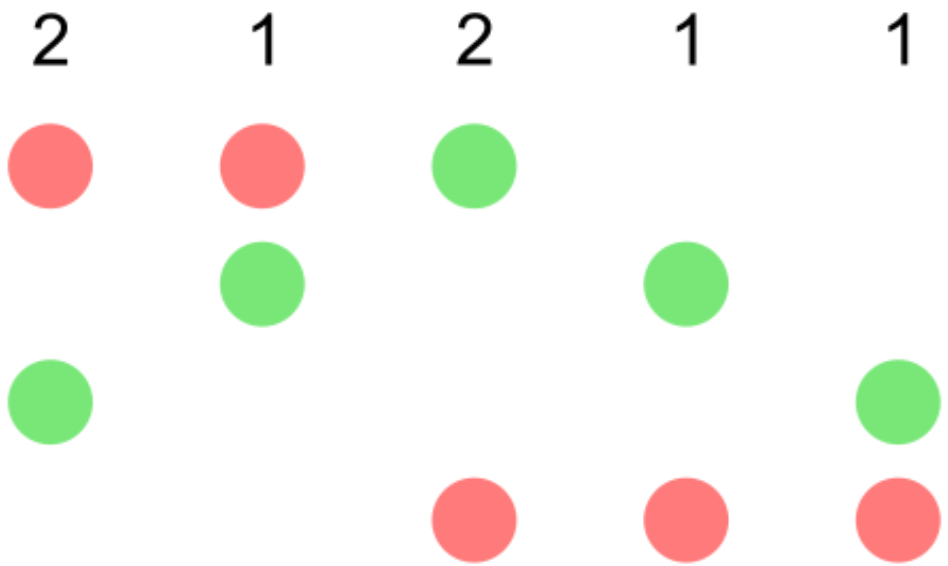

Plurality with runoff winner:

4

Single Transferable Vote

(Instant-Runoff)

4

In each round, eliminate the candidate with the lowest Plurality score, and transfer its supporters' votes

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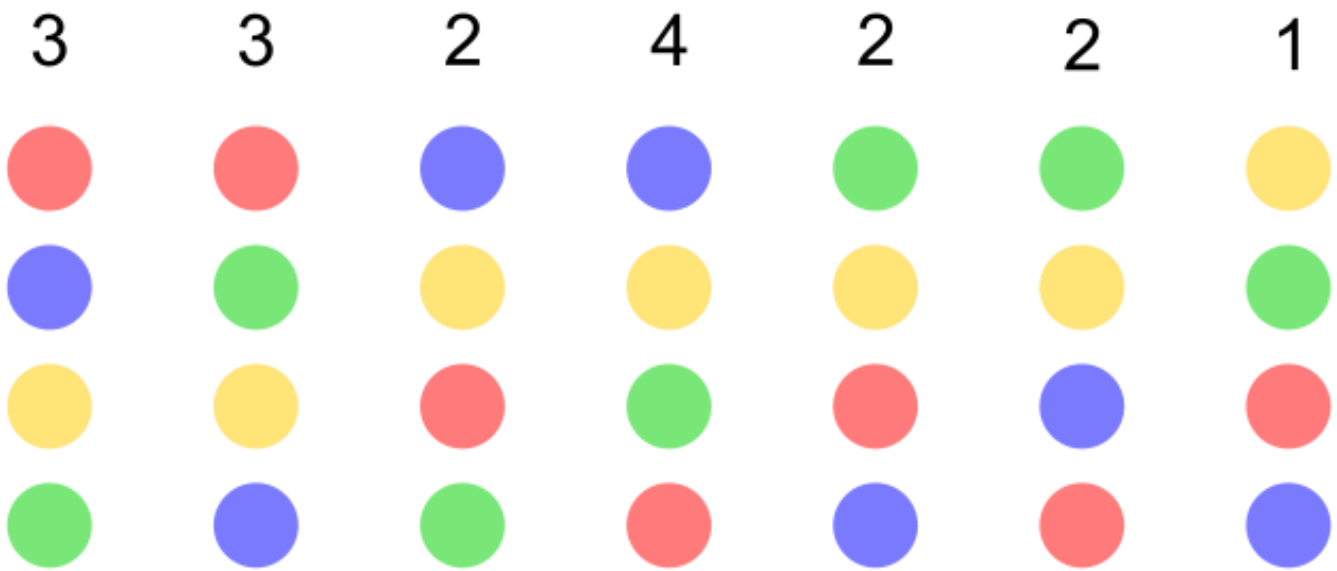

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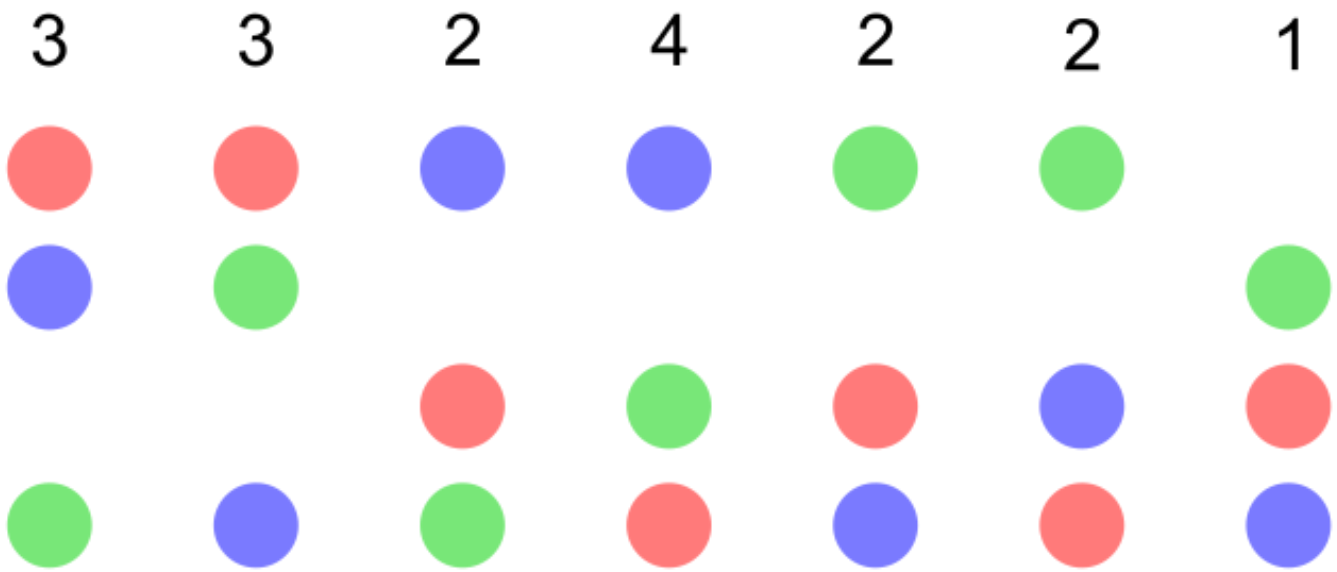

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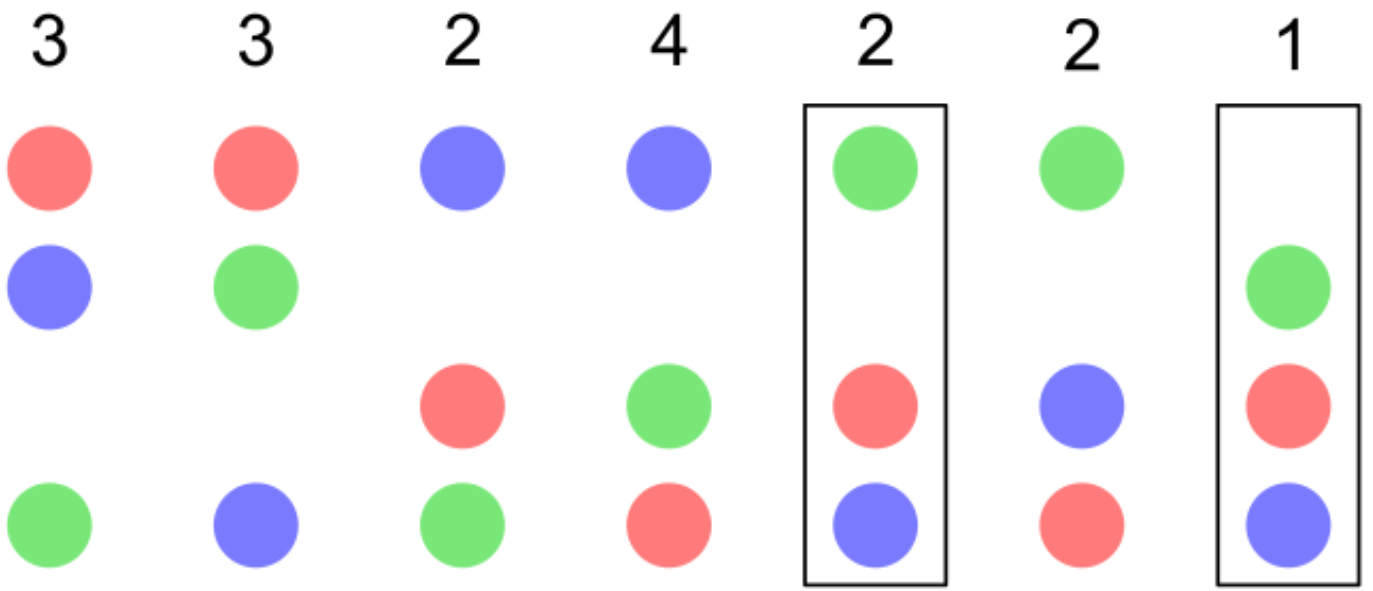

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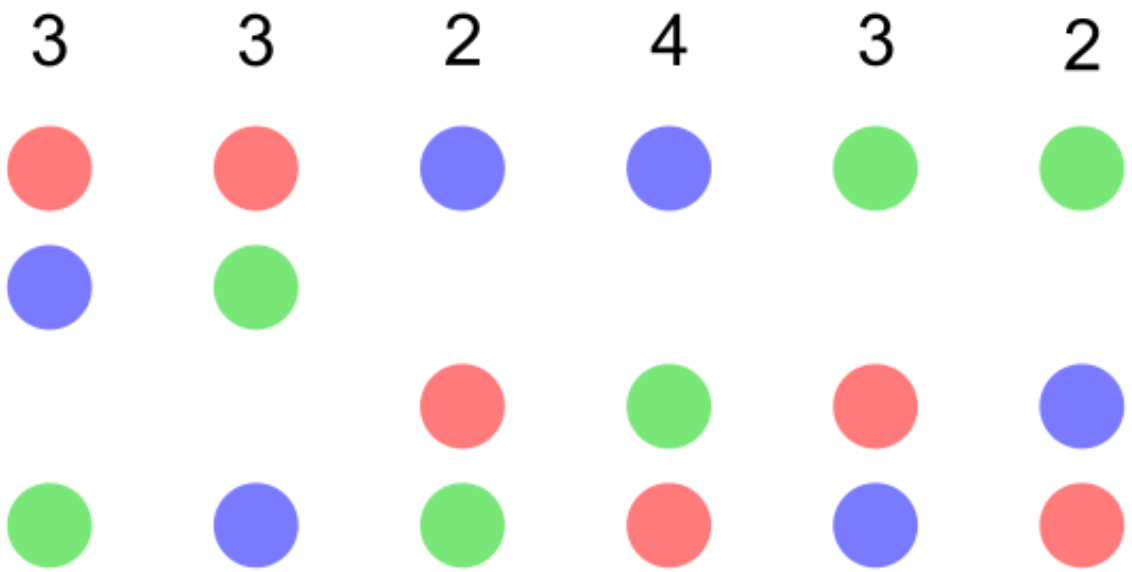

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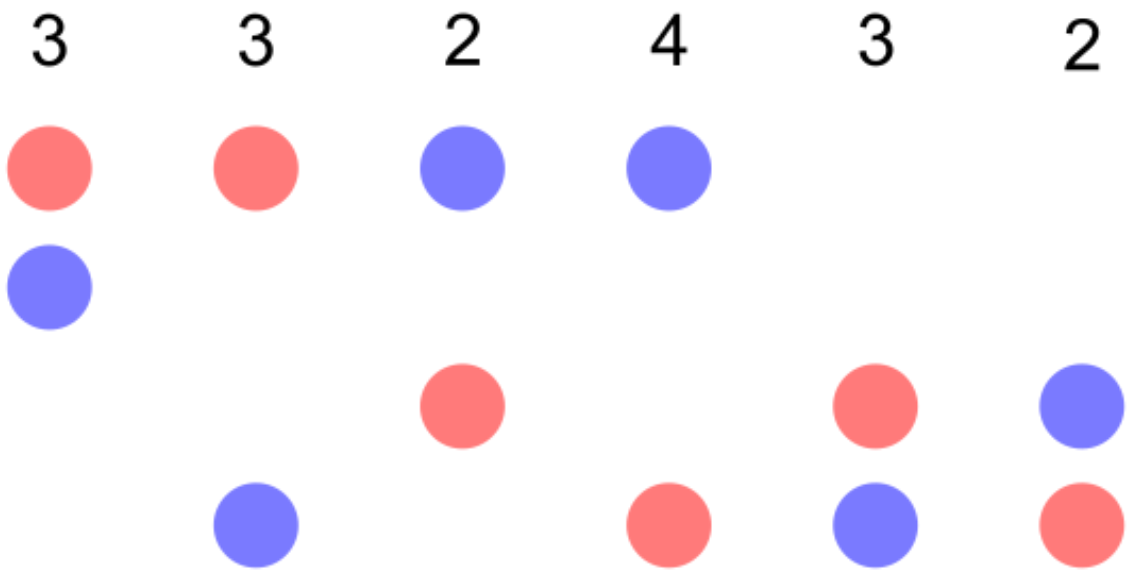

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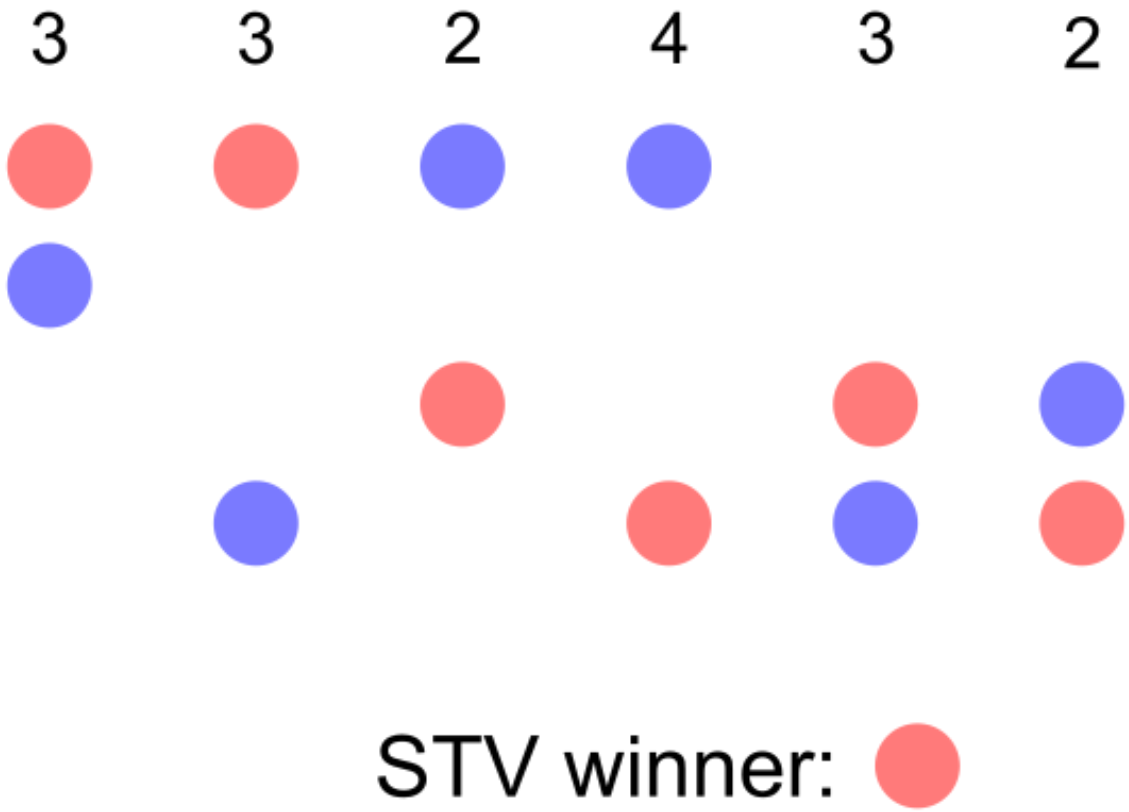

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Problem: Failure of *monotonicity*

(improving a candidate's support could make it worse off)

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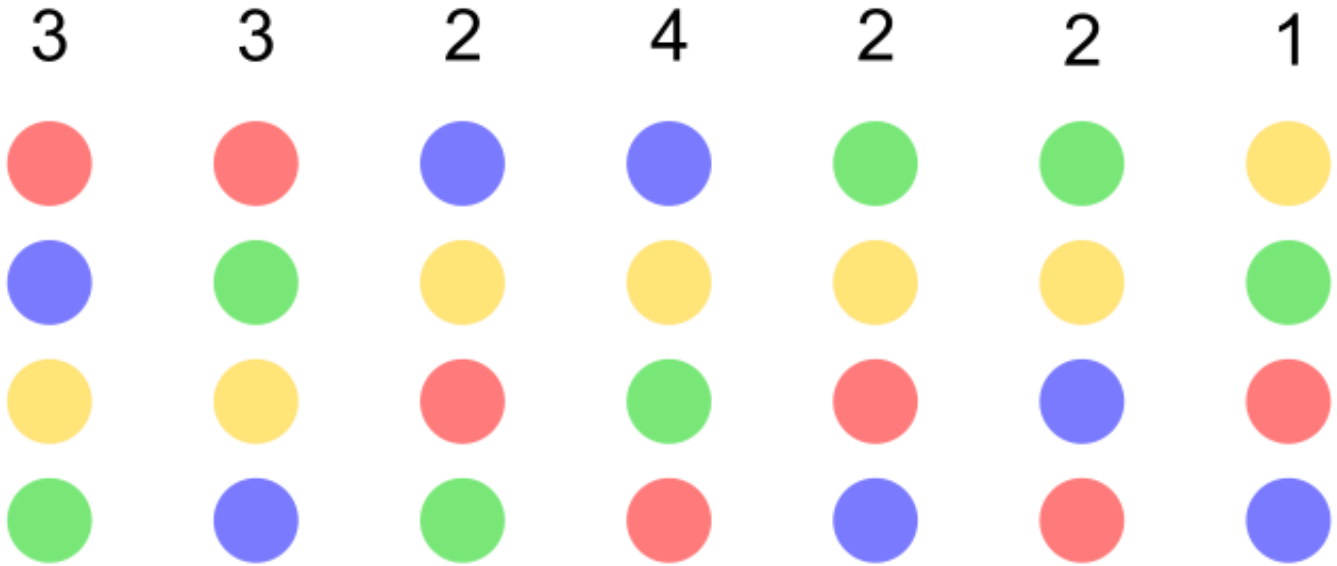

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Single Transferable Vote

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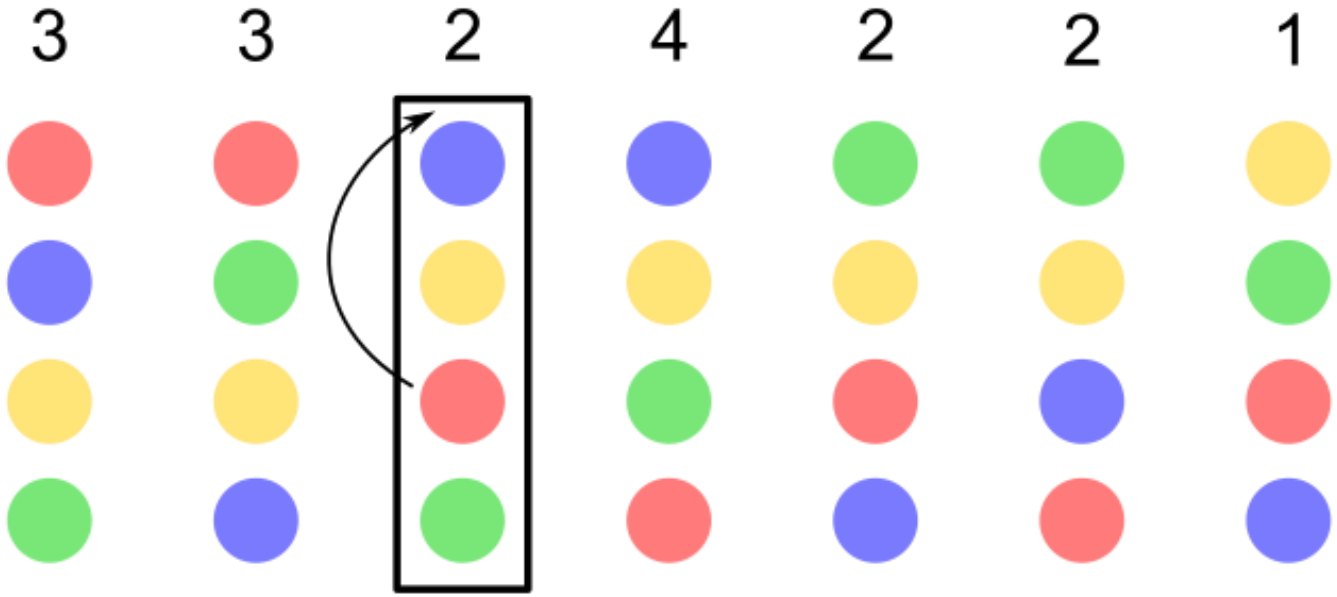

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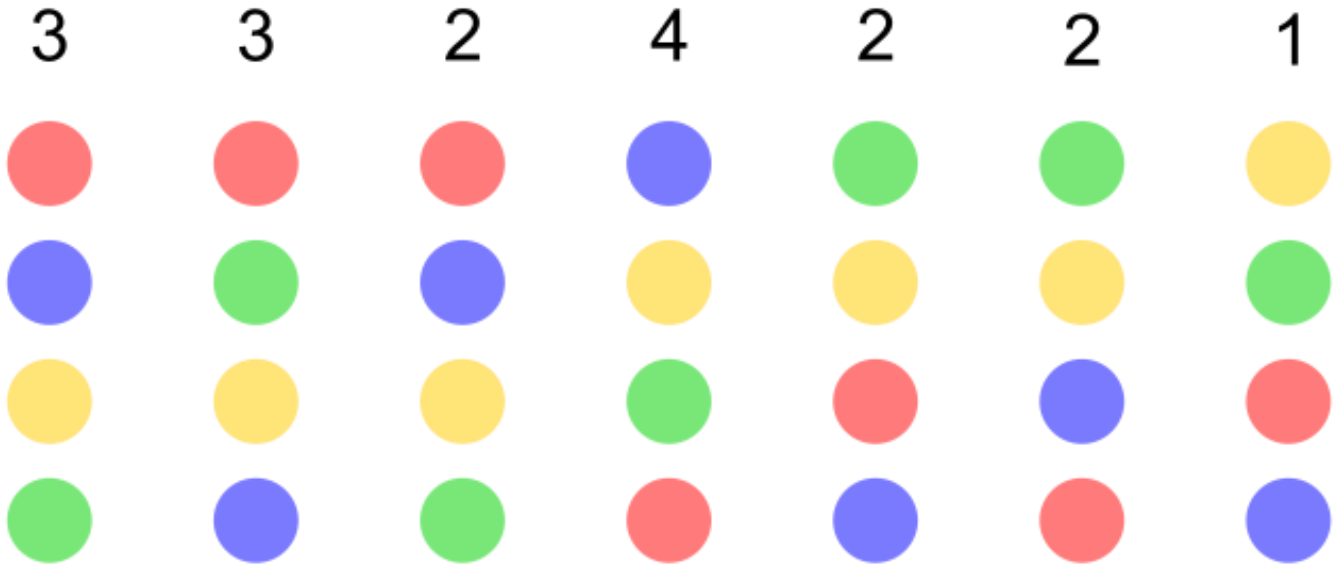

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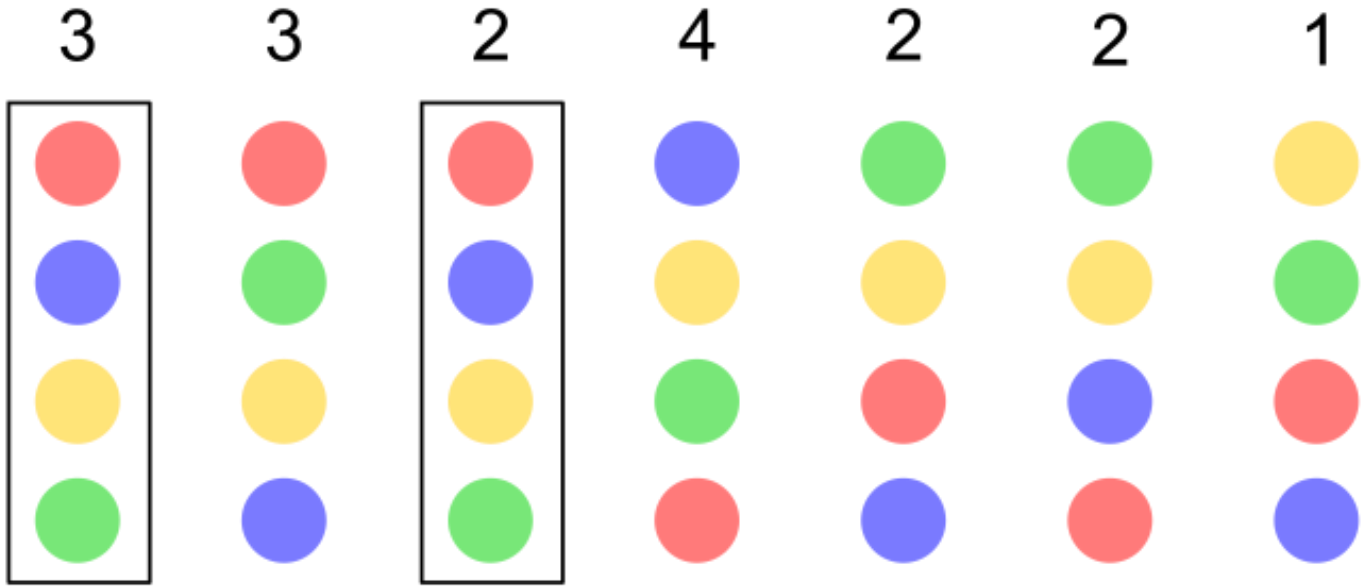

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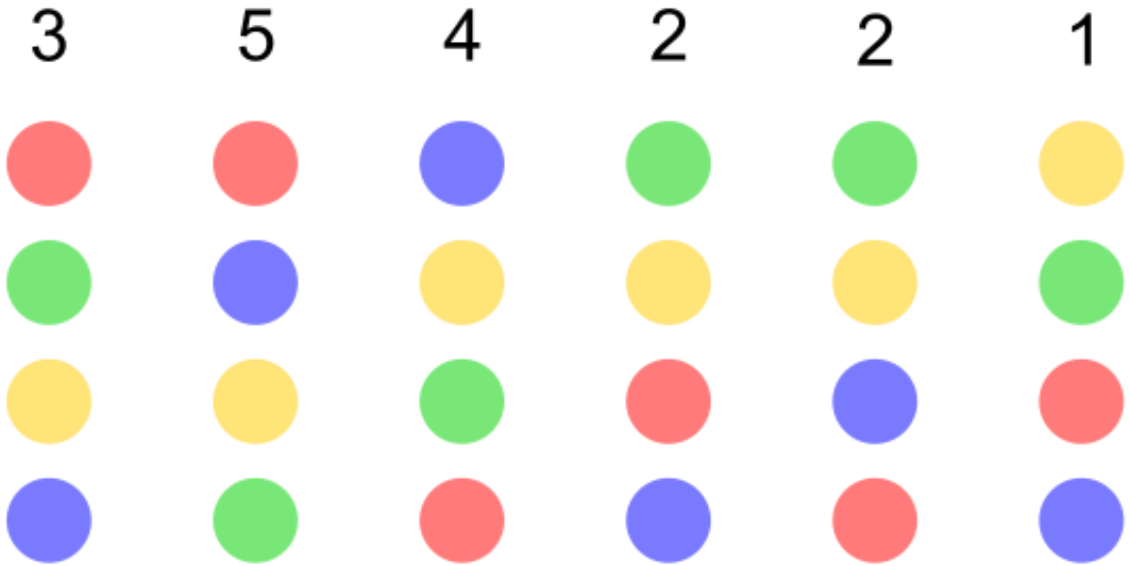

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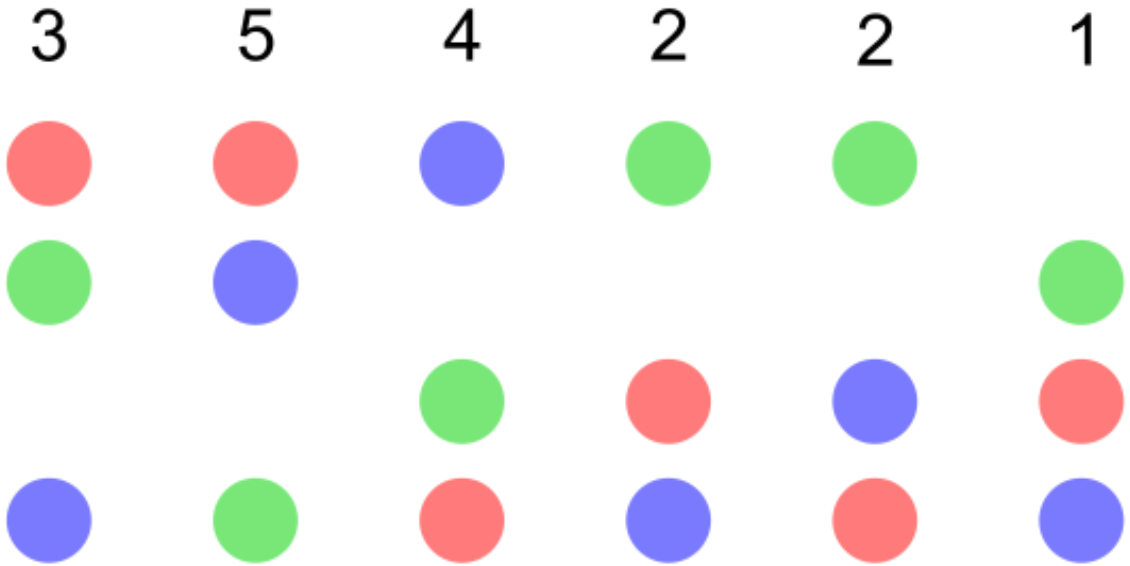

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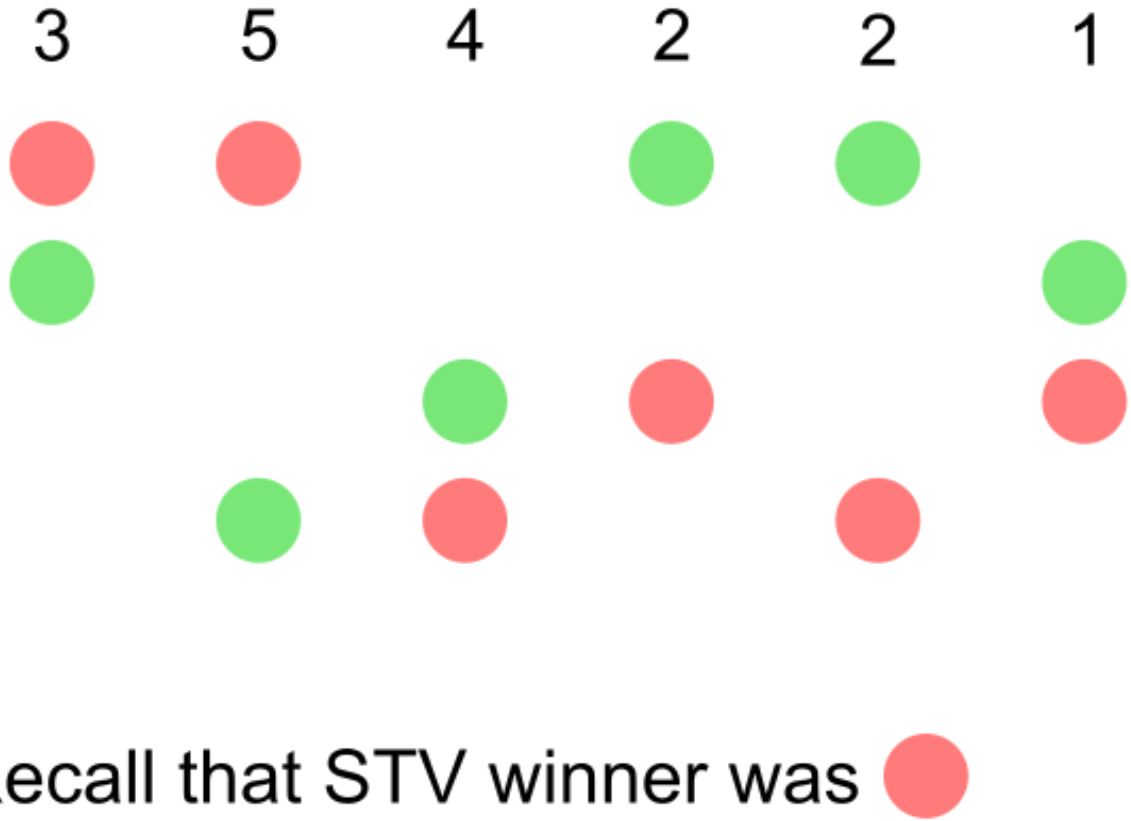

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2016 host city election ballots results [edit]

City	NOC	Round 1	Round 2	Round 3
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Madrid	Spain (COE)	**28**	29	32
Tokyo	Japan (JOC)	22	20	—
Chicago	United States (USOC)	18	—	—

Venue
Bella Center **121st IOC Session** October 2, 2009 Copenhagen

Vote details	Round 1	Round 2	Round 3
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Participants	94	96	98
Abstentions	0	1	0
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Nicolas de Condorcet

(1743-1794)

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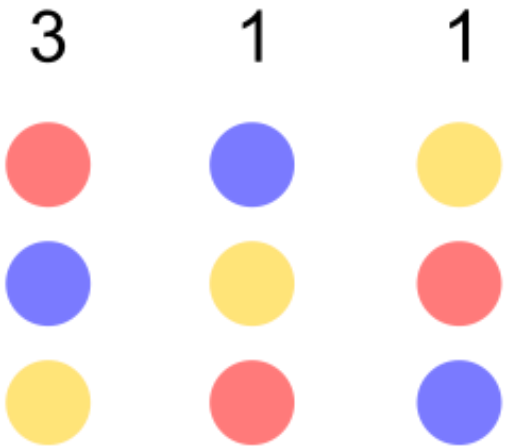

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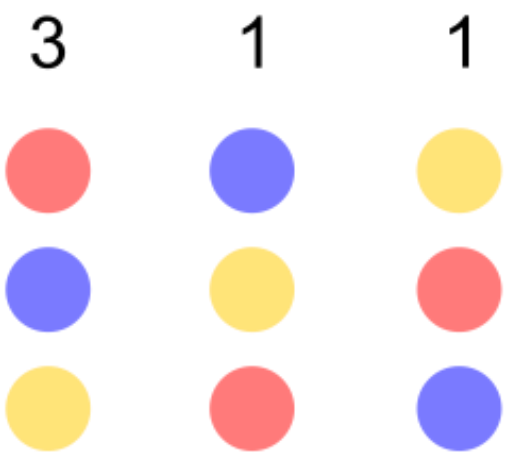

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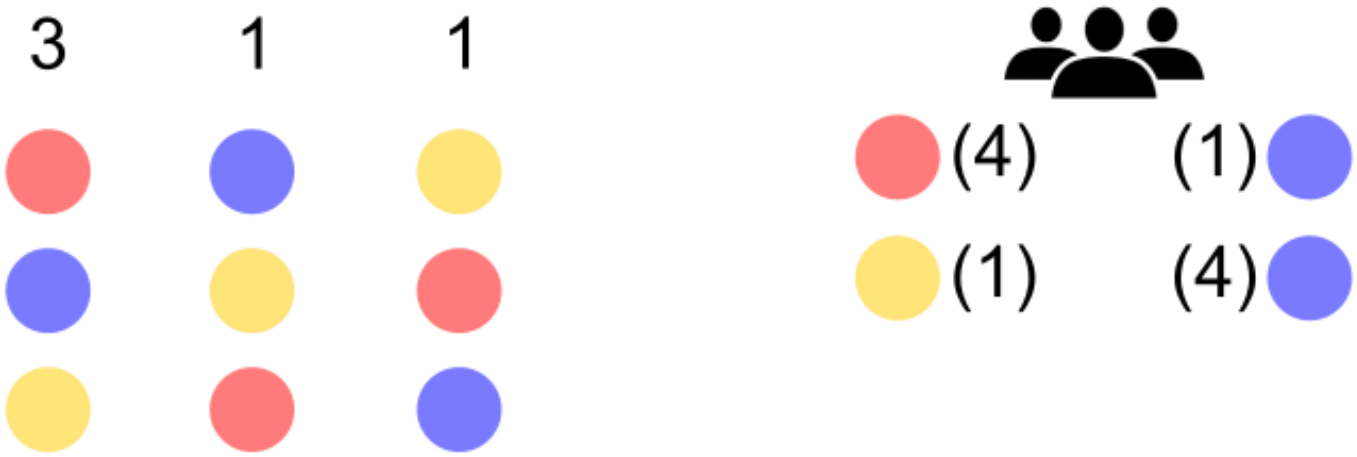

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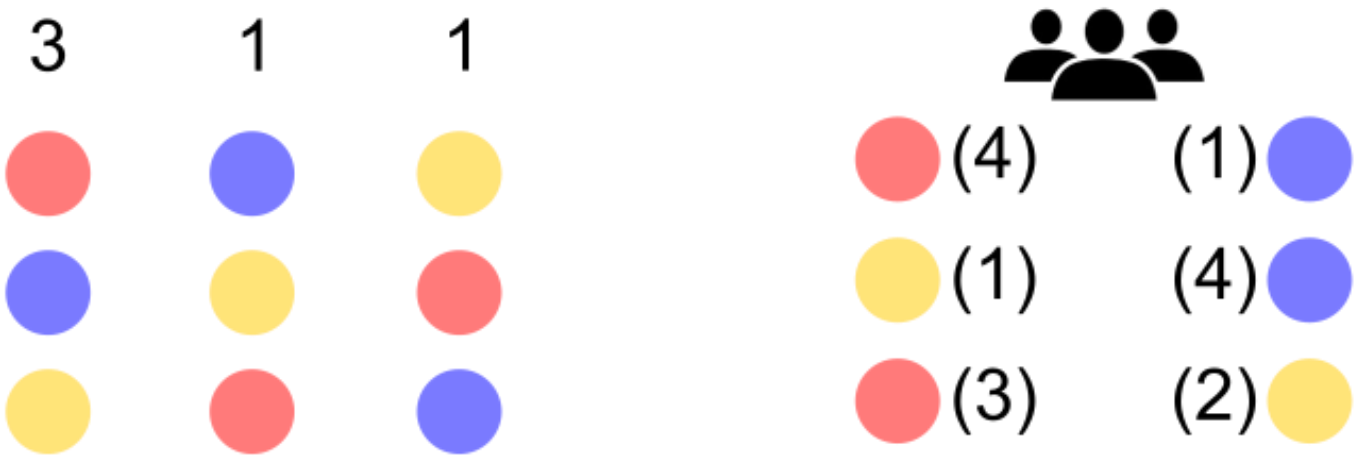

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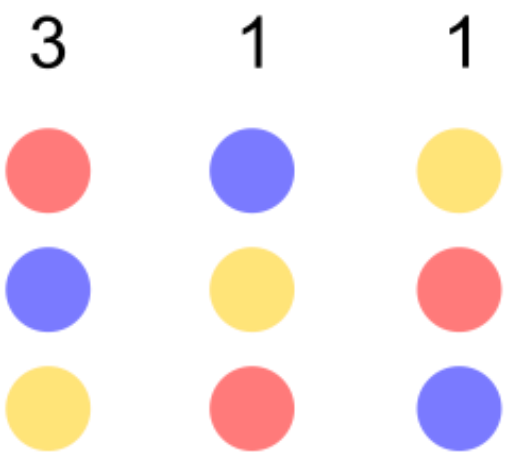

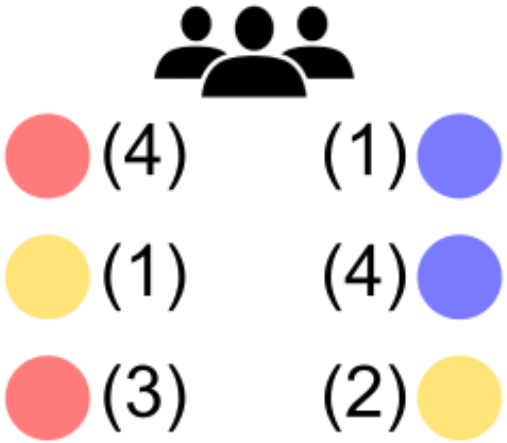

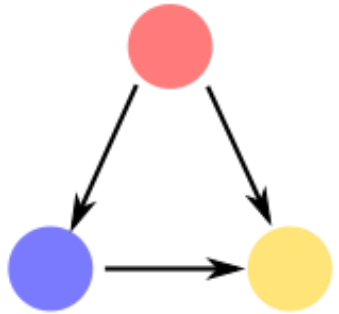

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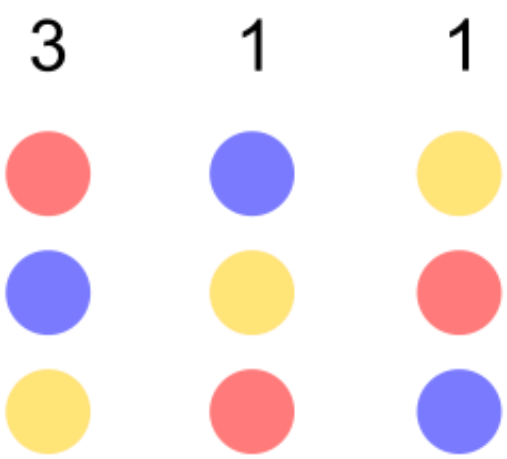
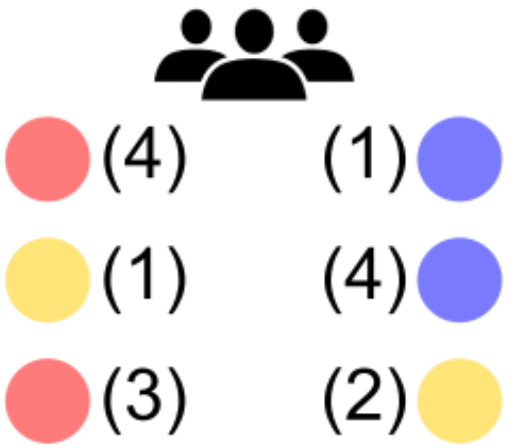
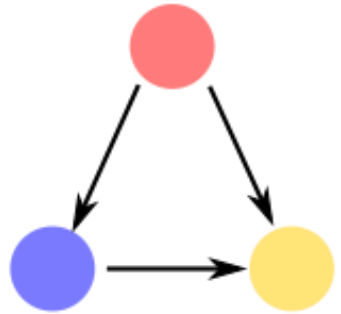




Condorcet winner



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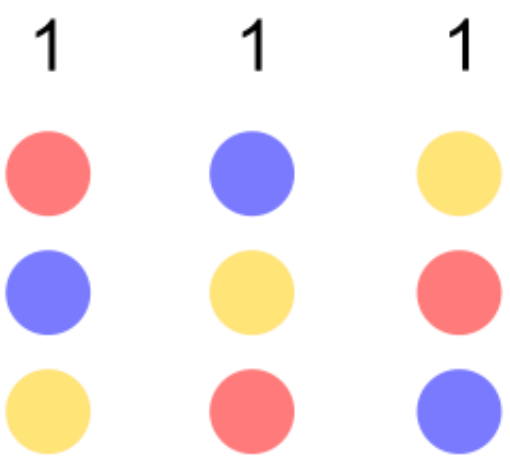

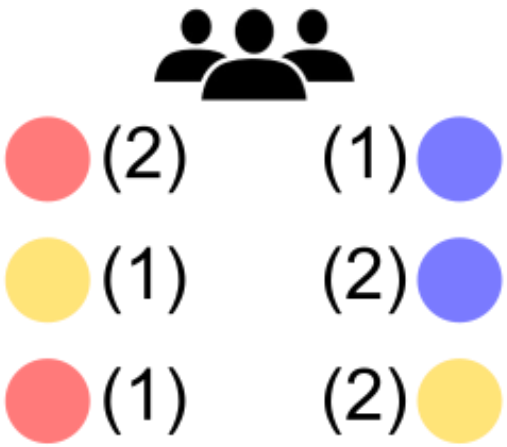

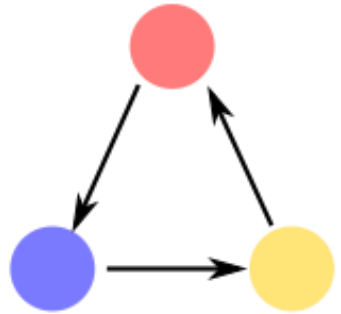

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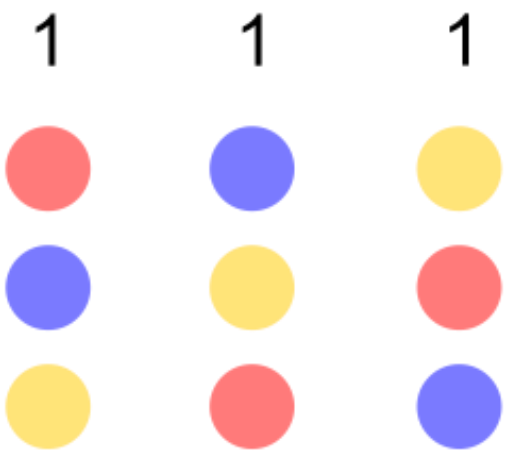

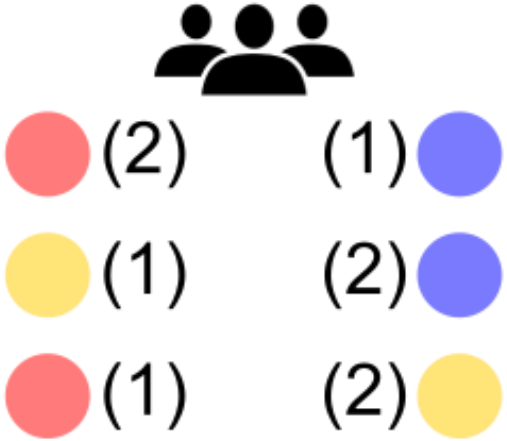

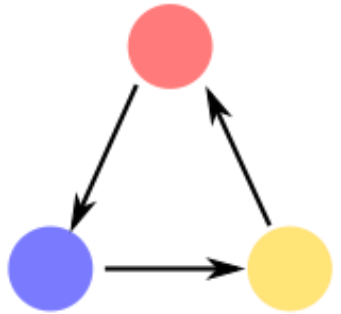

Condorcet paradox

Transitivity of individual preferences ⇏ Transitivity of societal preferences

5

Copeland

5

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Copeland

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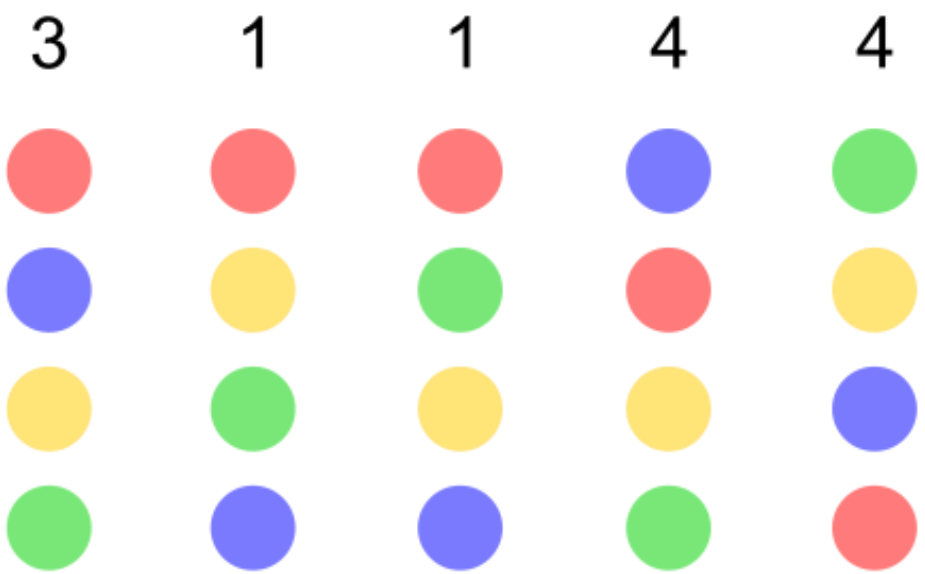

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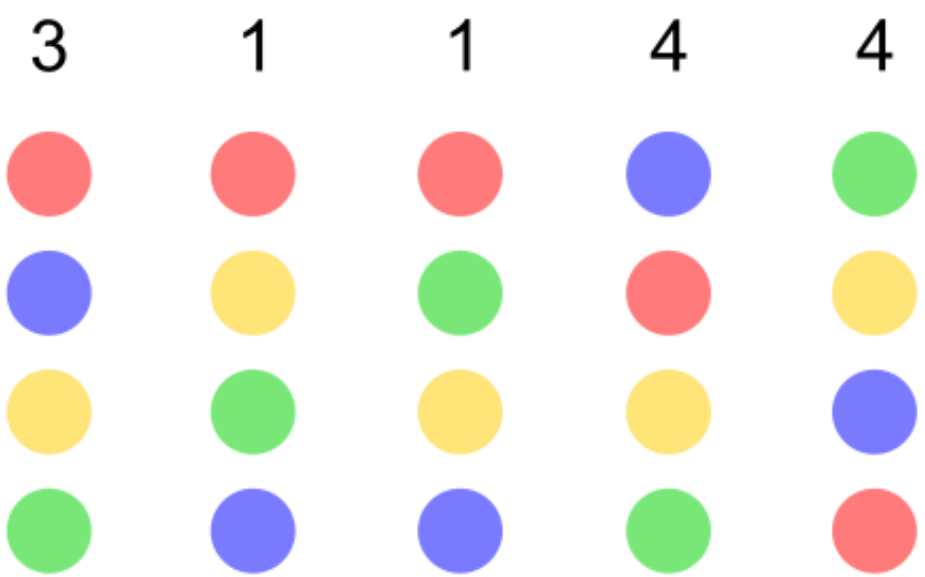

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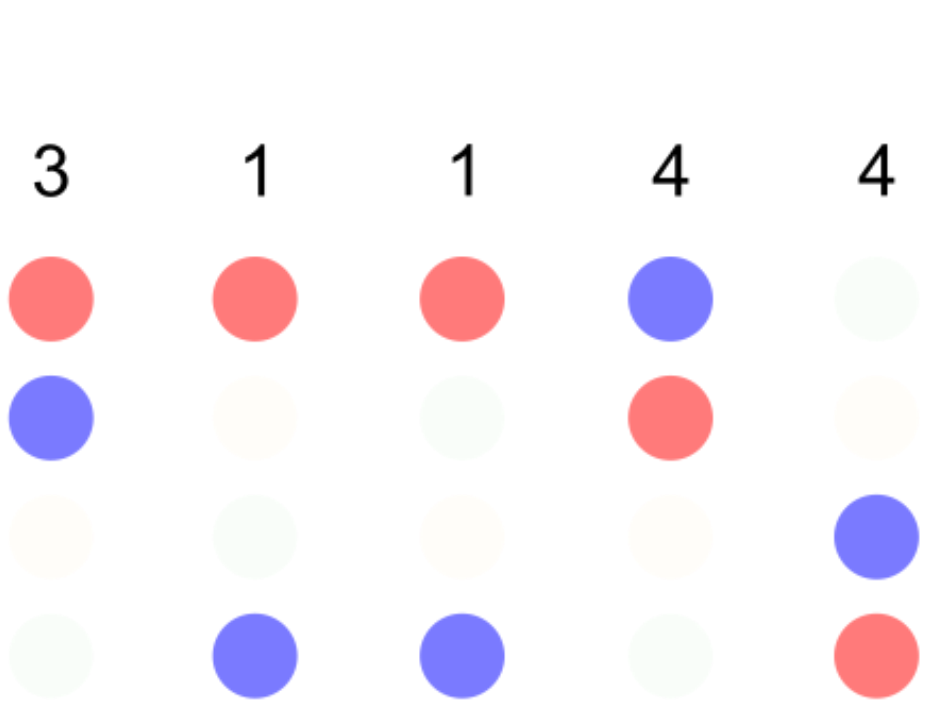

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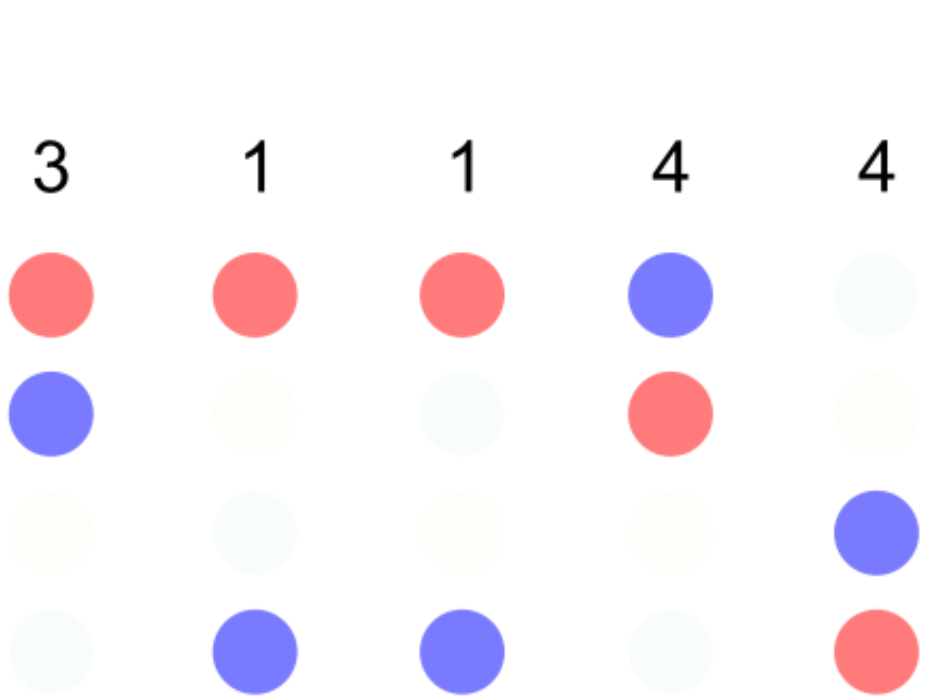

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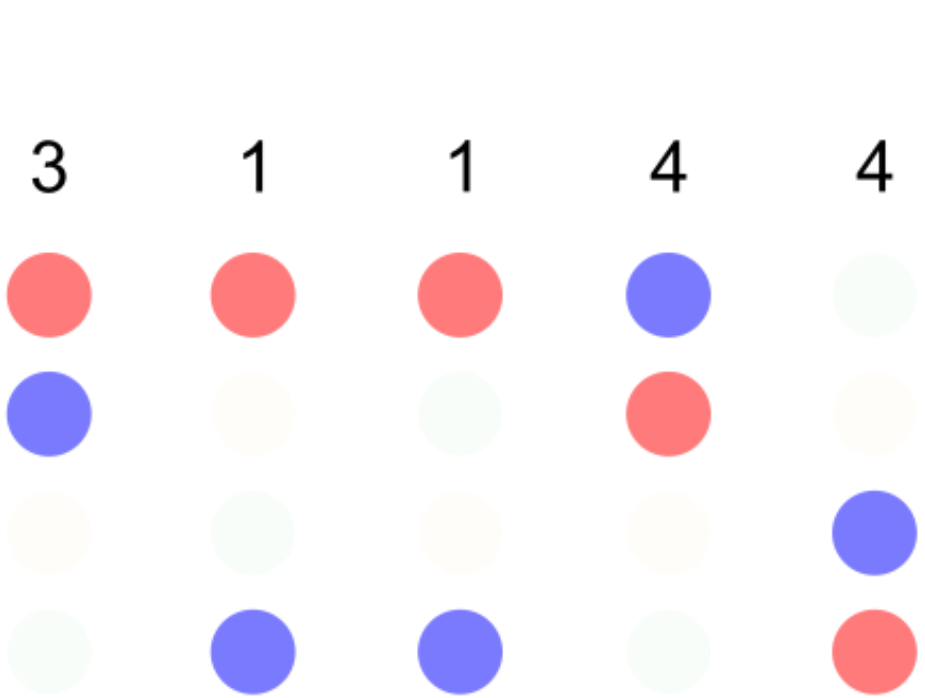

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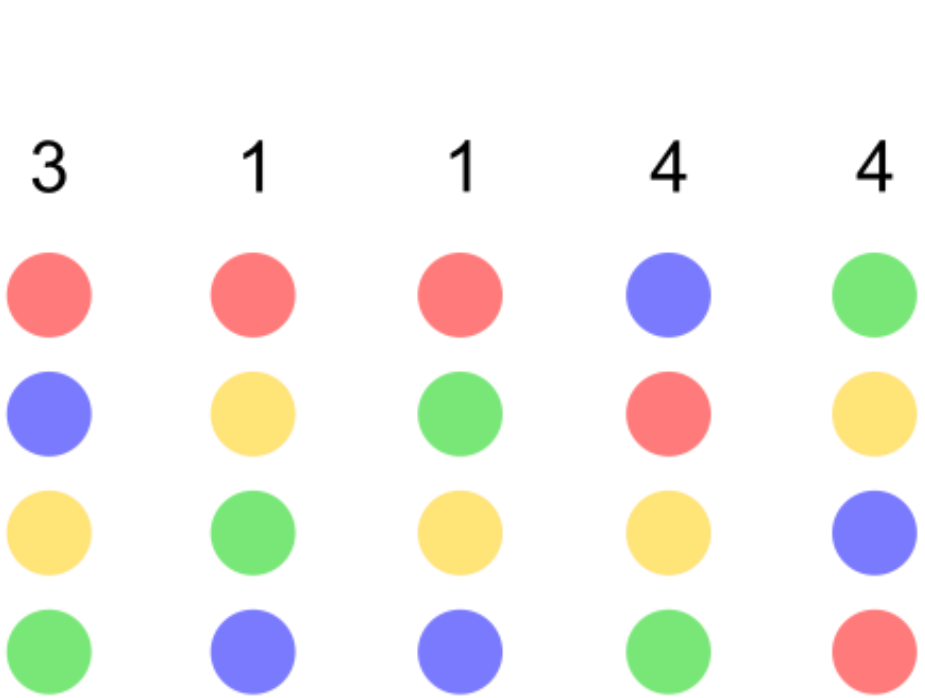

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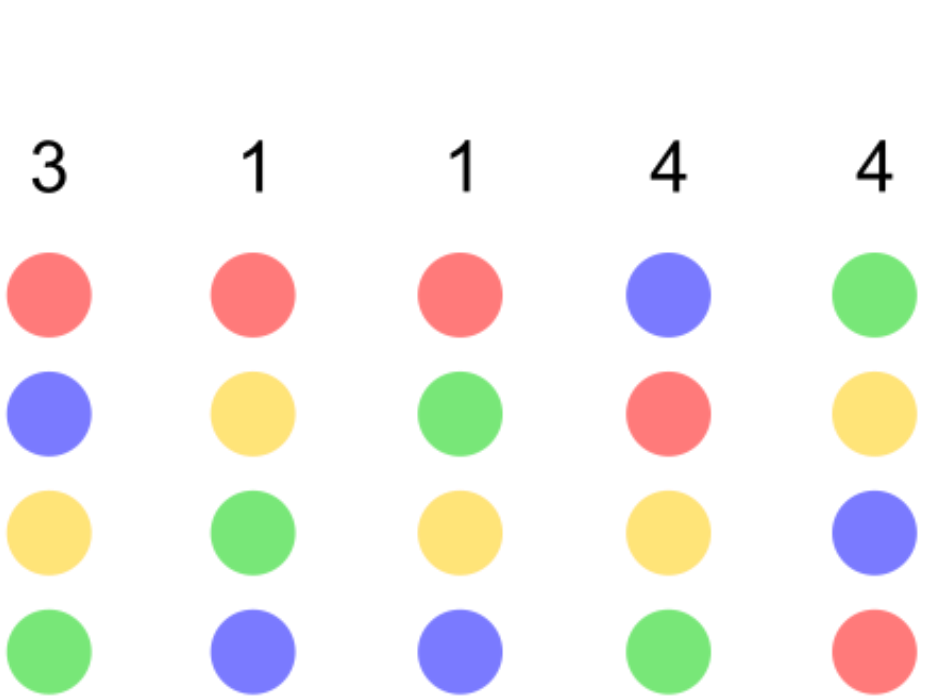

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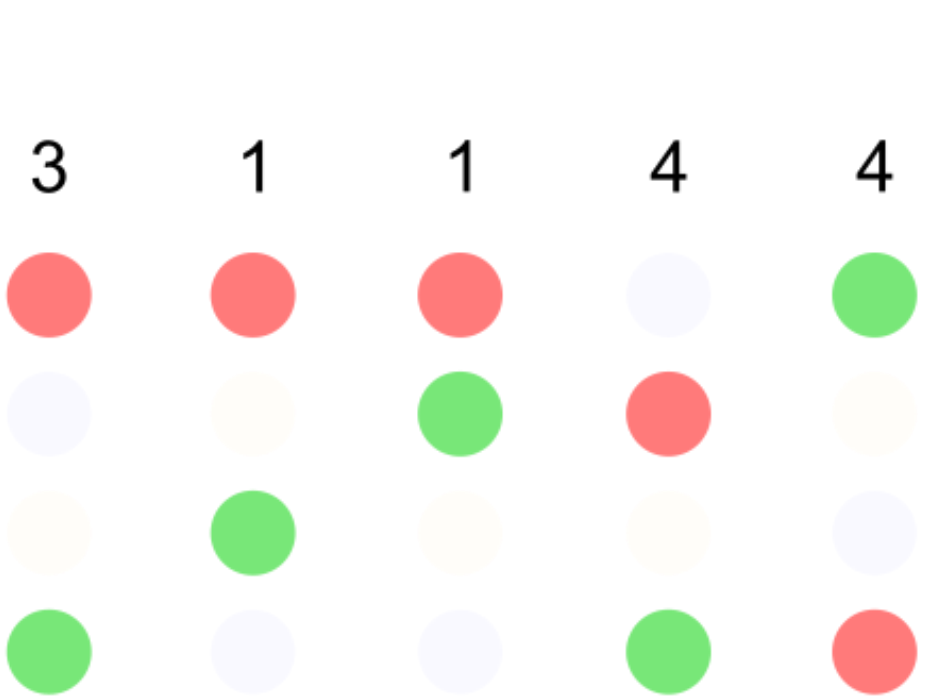

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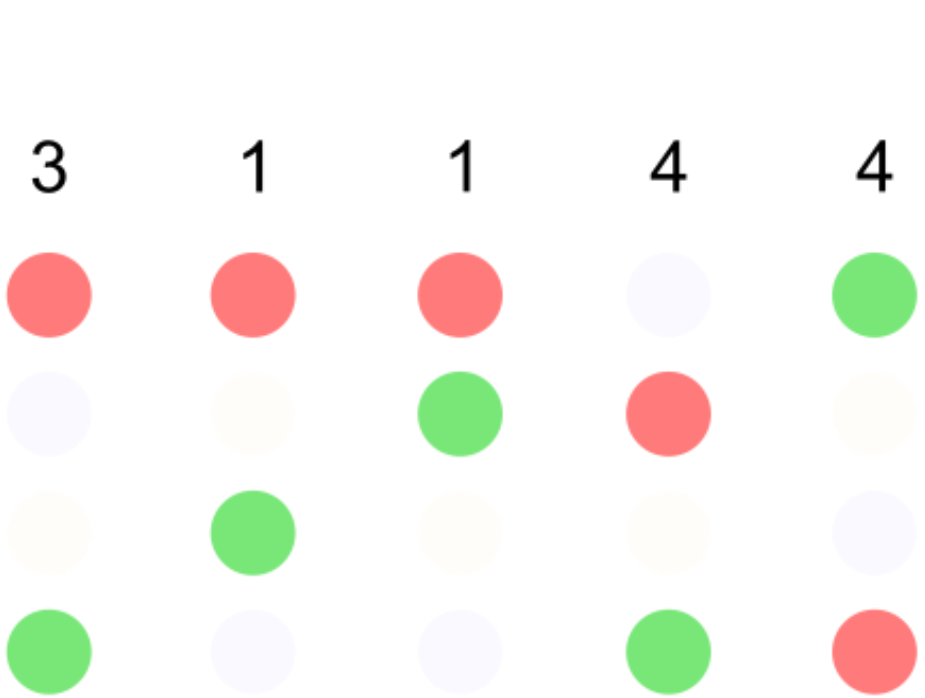

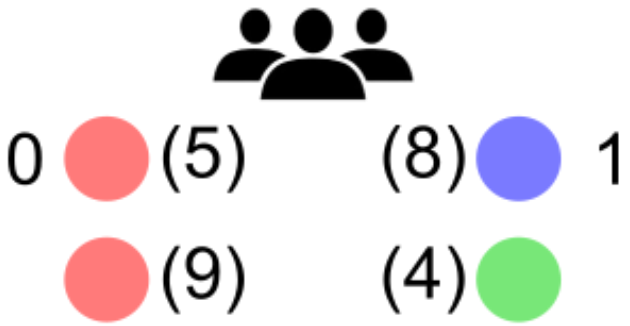

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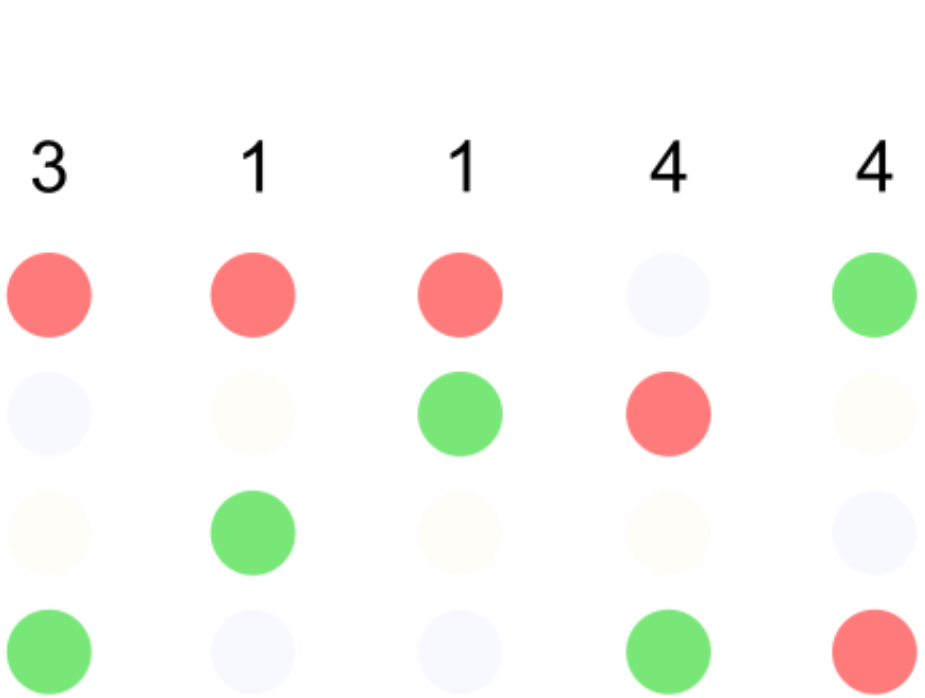

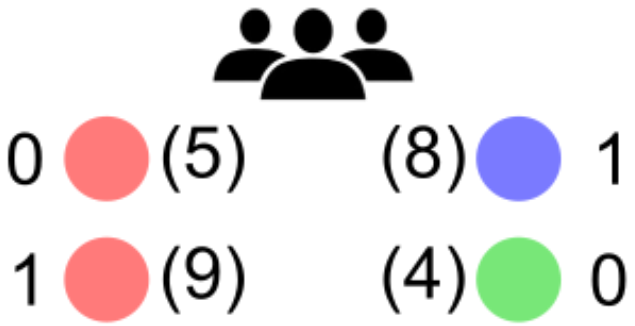

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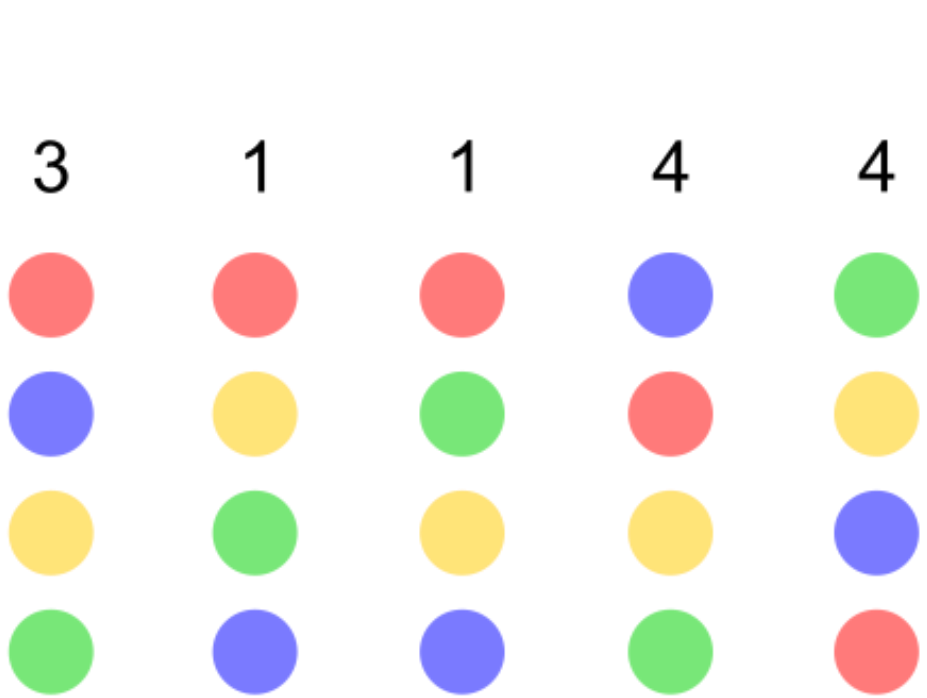

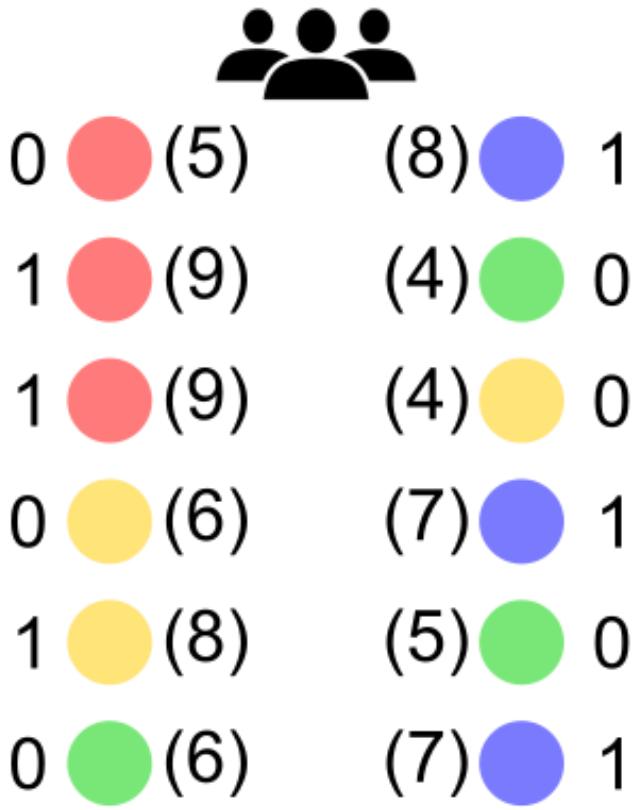

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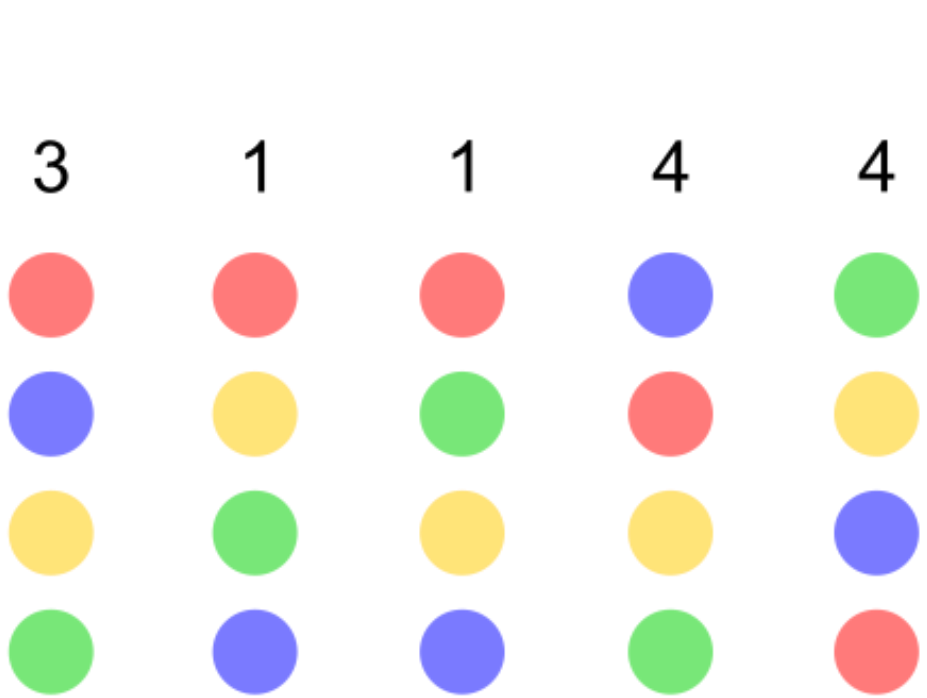

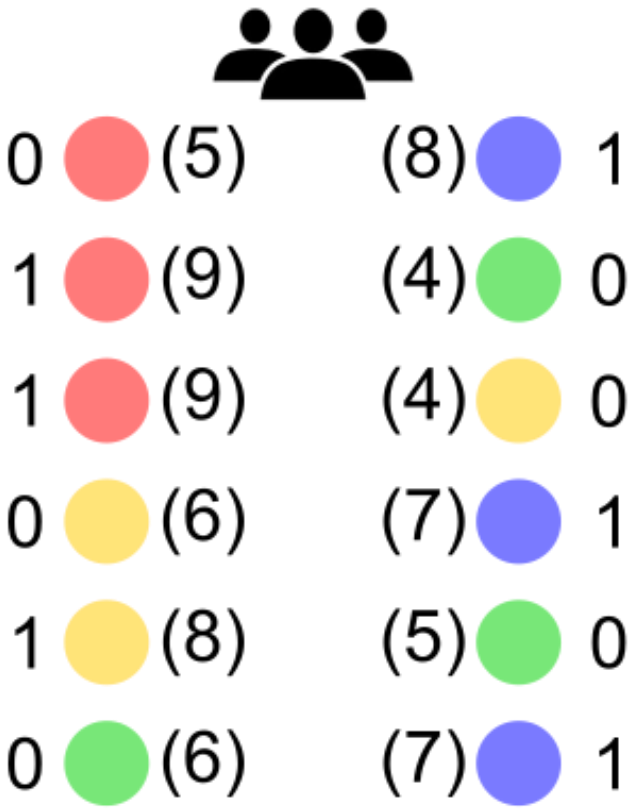

Copeland winner:

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Copeland

LaLiga

BUNDESLIGA

ICC WOMEN'S CRICKET WORLD CUP NEW ZEALAND 2022

Premier League

INDIAN PREMIER LEAGUE ®

5

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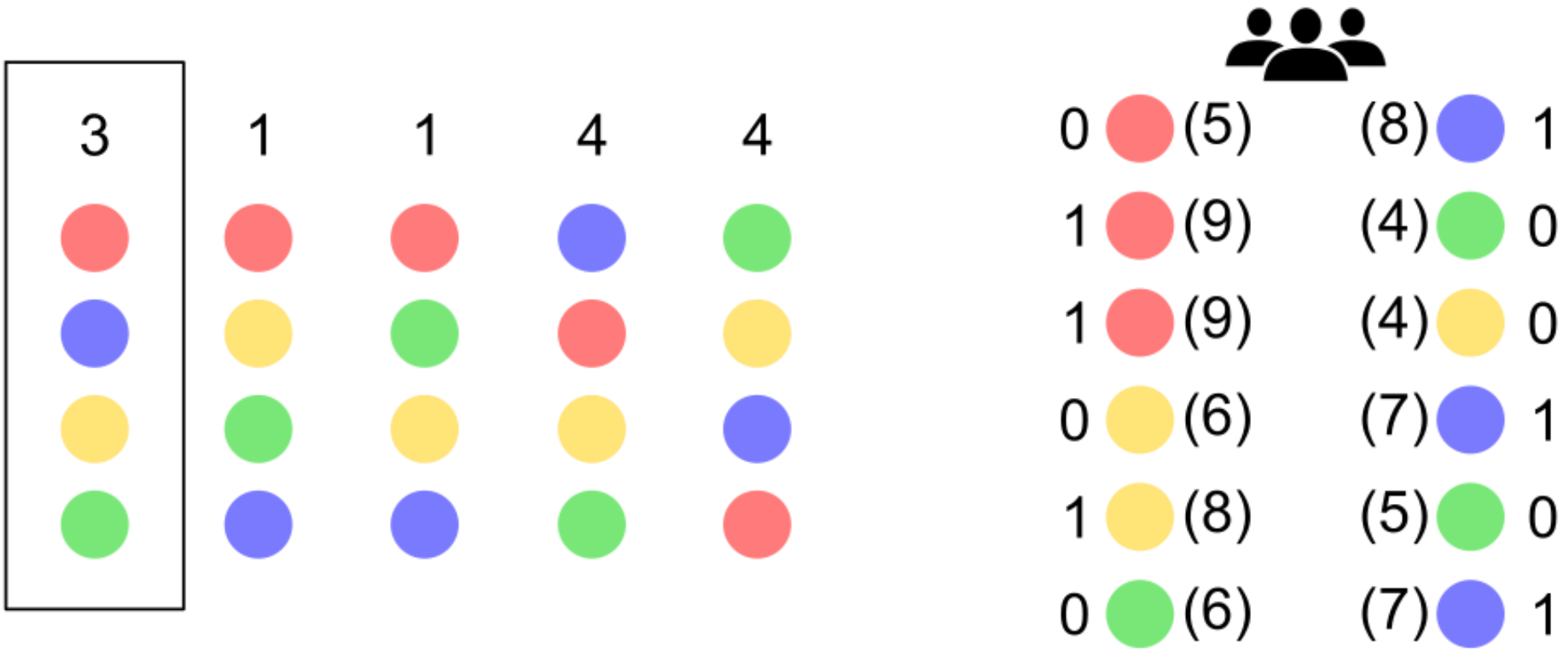

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Copeland winner:

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Schulze

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Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph

Schulze

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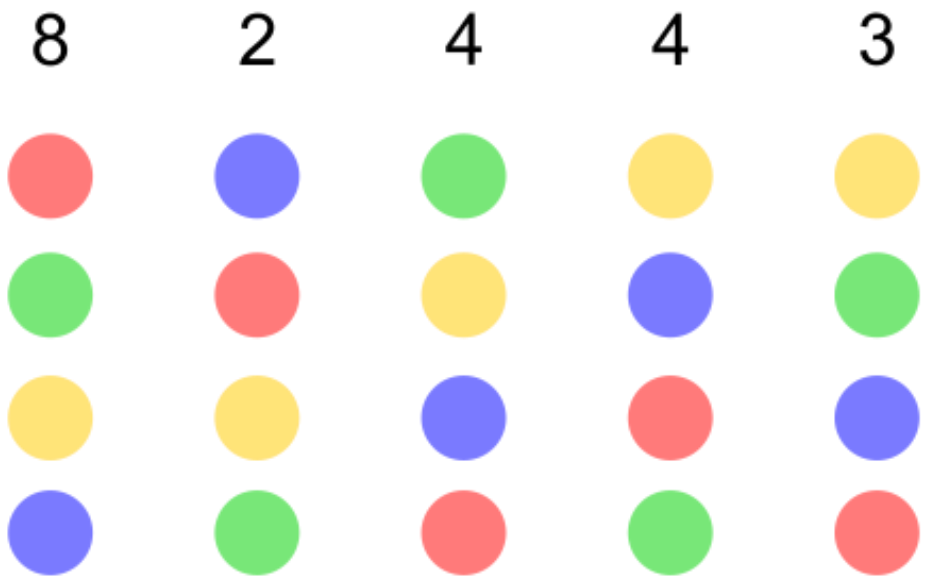
Schulze



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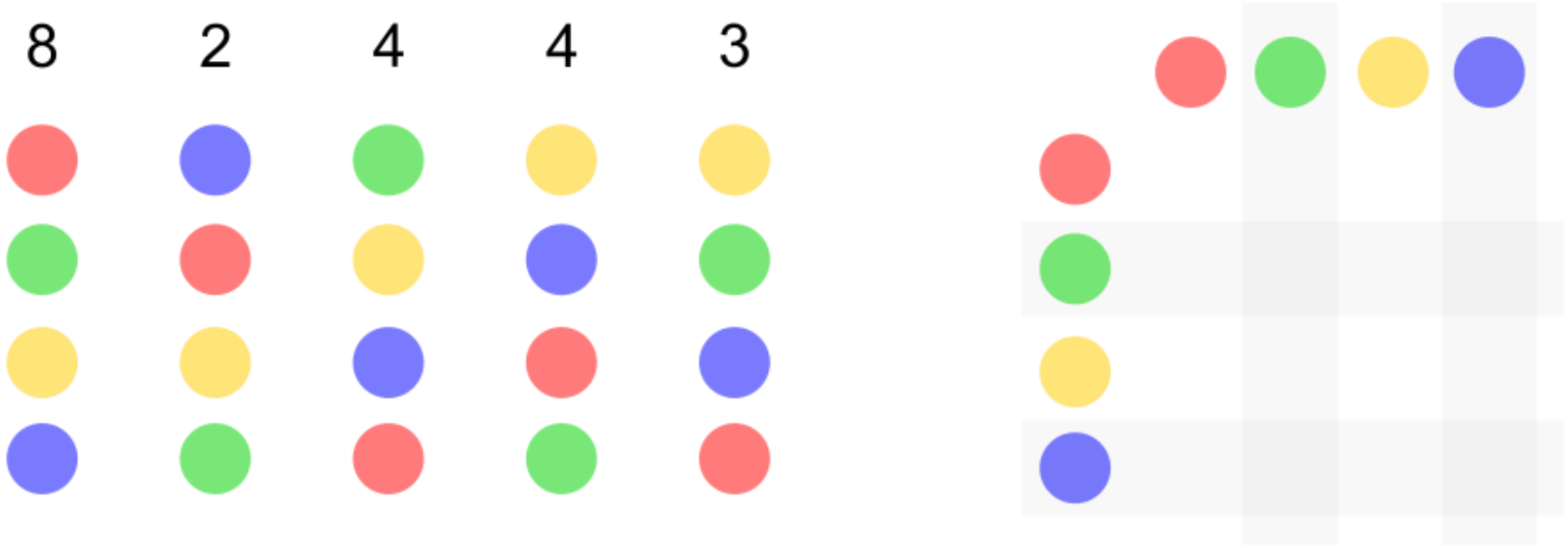

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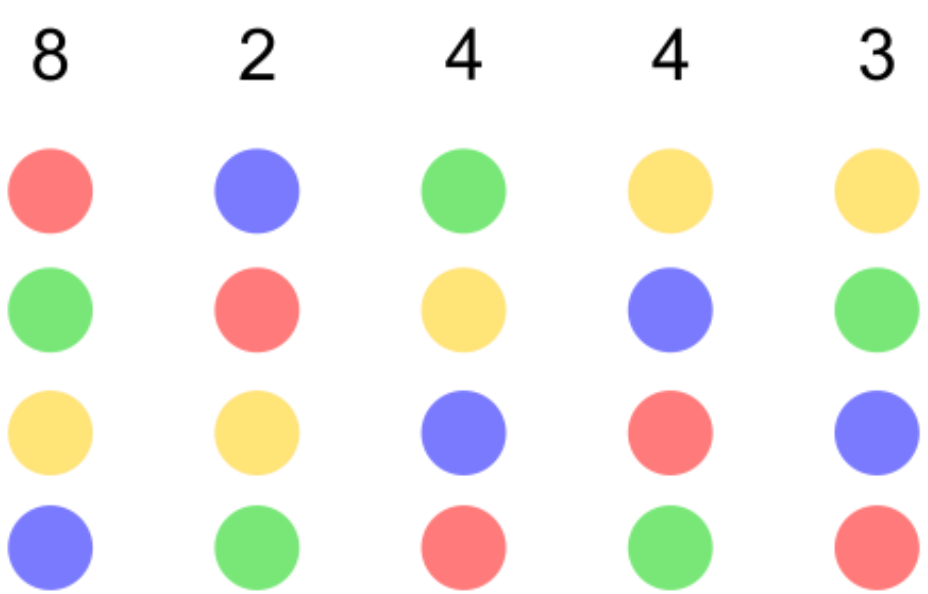

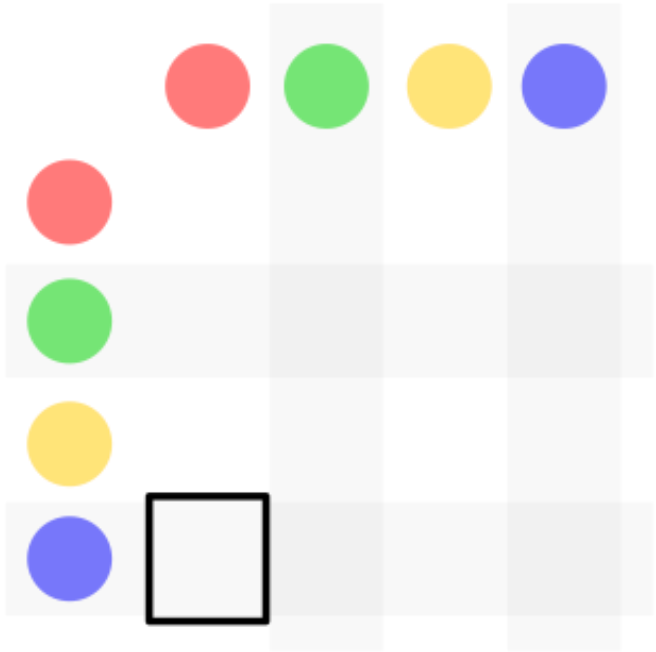

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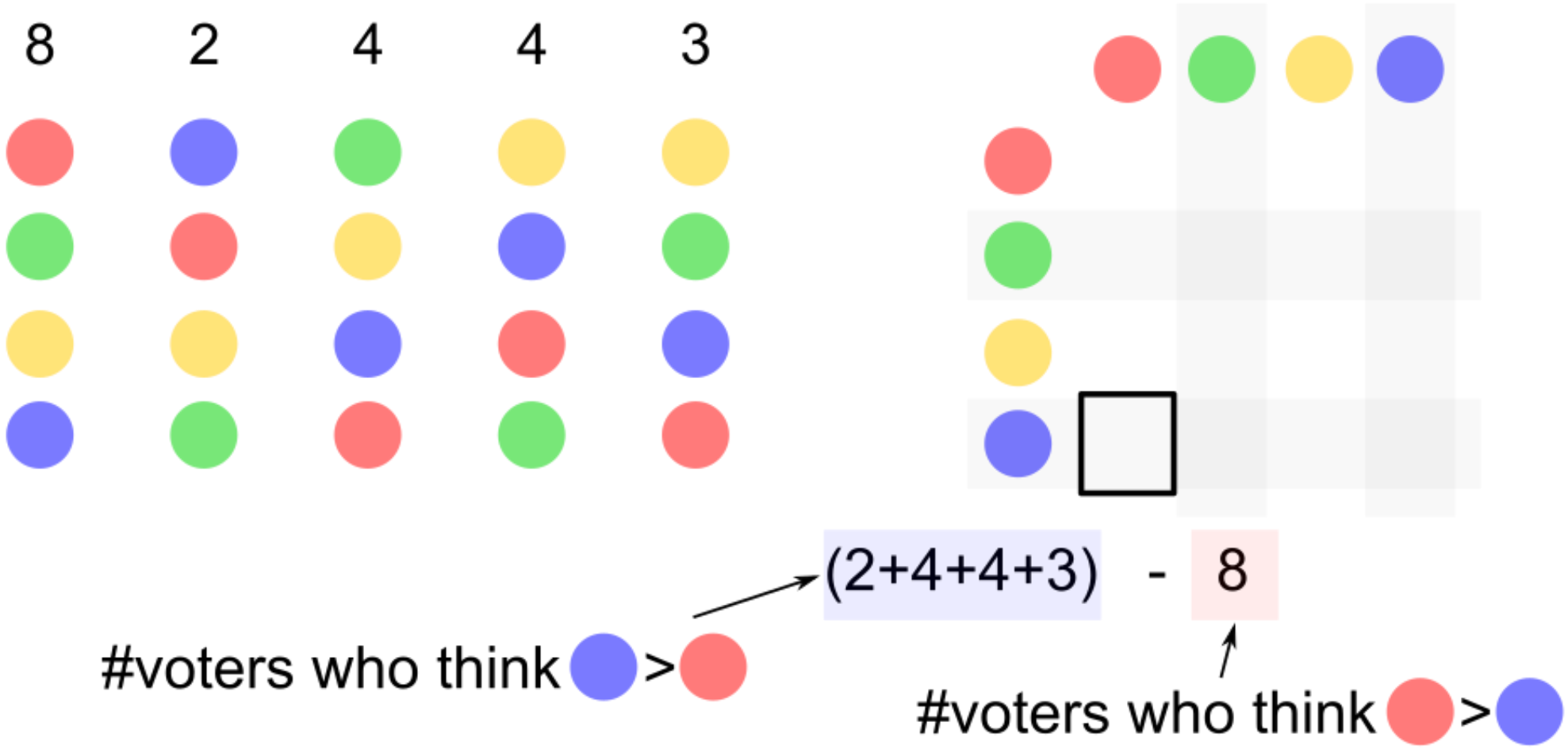

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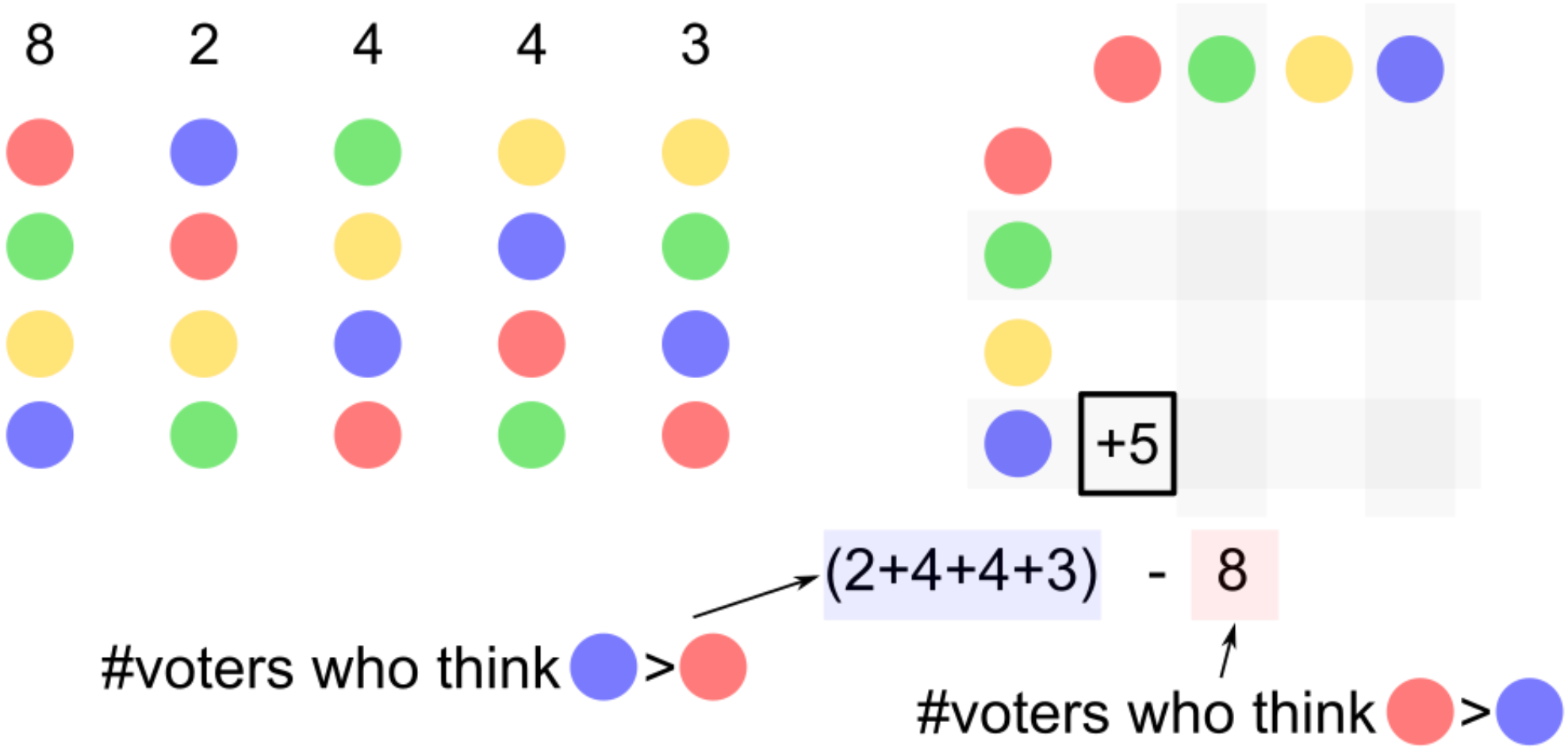

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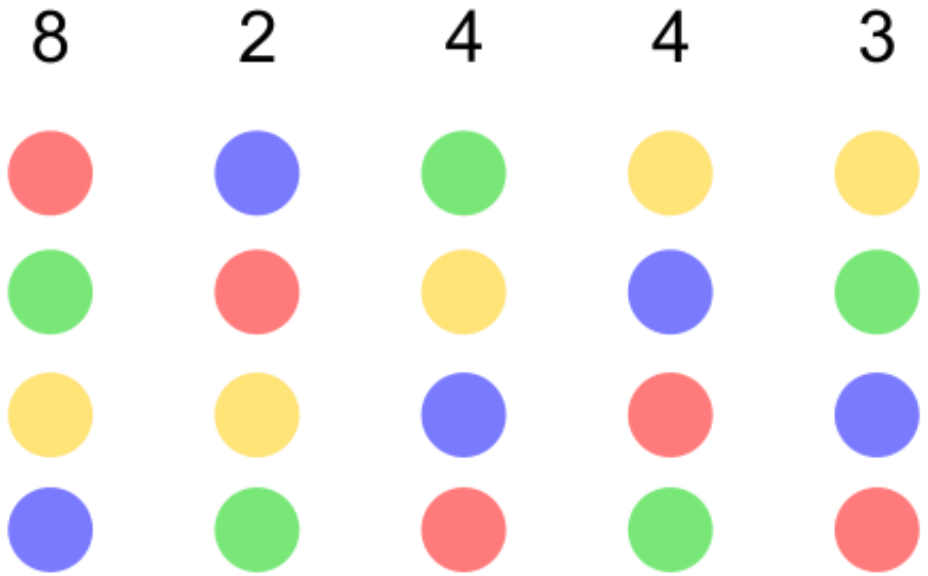

	0	+7	-1	-5
	-7	0	+3	+9
	+1	-3	0	+17
	+5	-9	-17	0

#voters who think >

(2+4+4+3) - 8

#voters who think >

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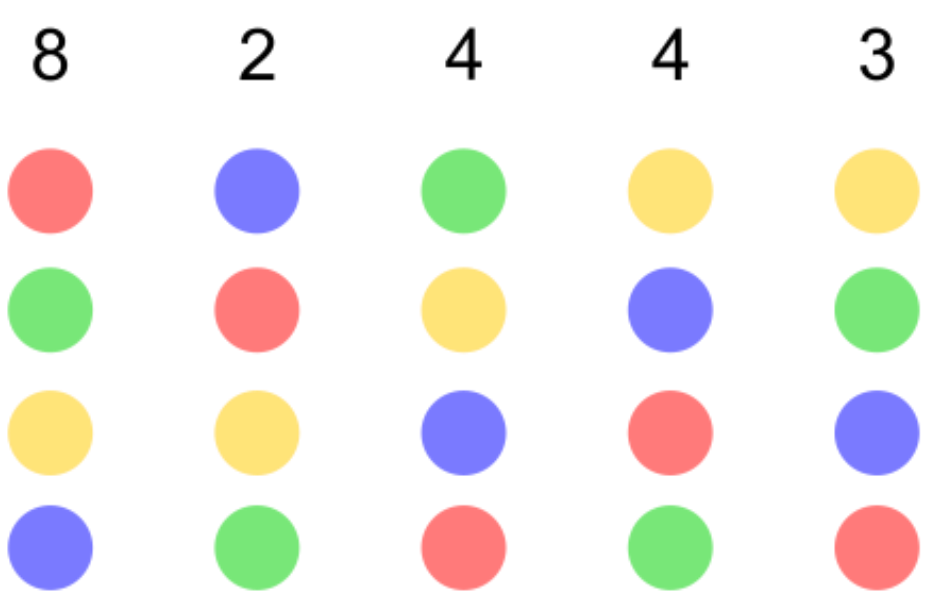
Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph

Schulze



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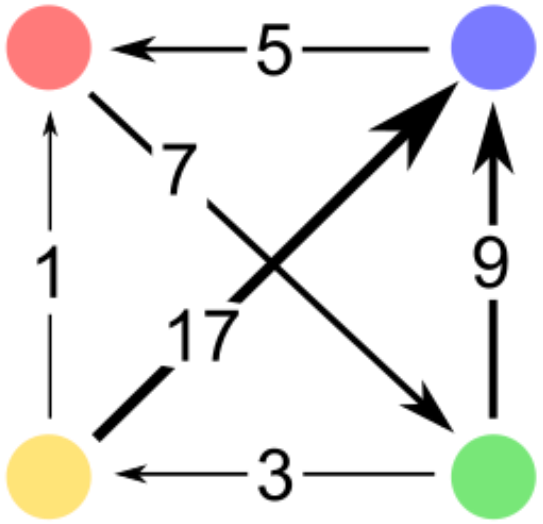
Schulze

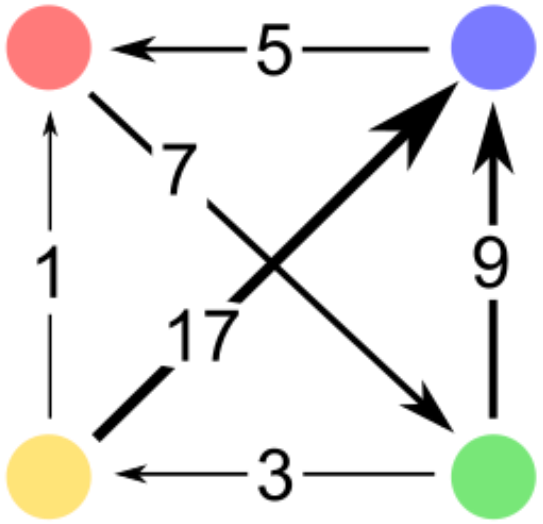


	0	+7	-1	-5
	-7	0	+3	+9
	+1	-3	0	+17
	+5	-9	-17	0

6

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph

Schulze



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Schulze

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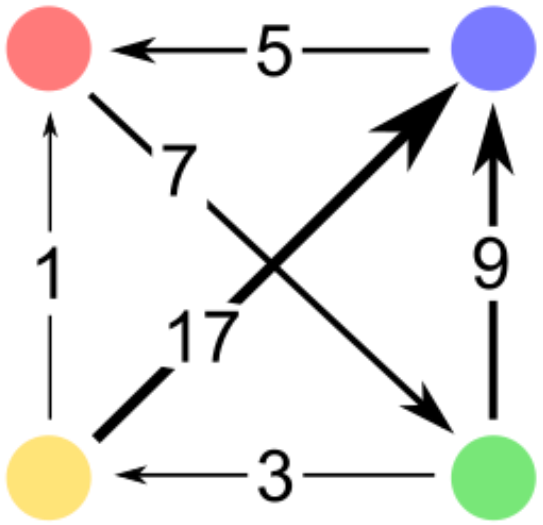

The strength of a path/chain is the weight of the weakest link in it

6

Schulze

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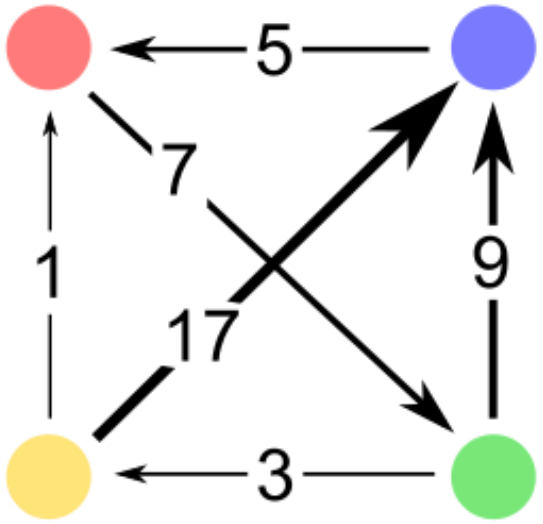

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What's the strongest path from ● to ●?

6

Schulze

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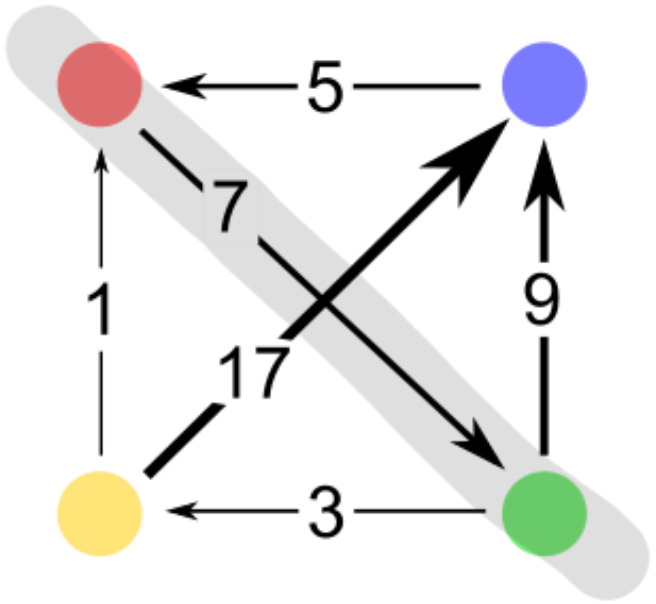

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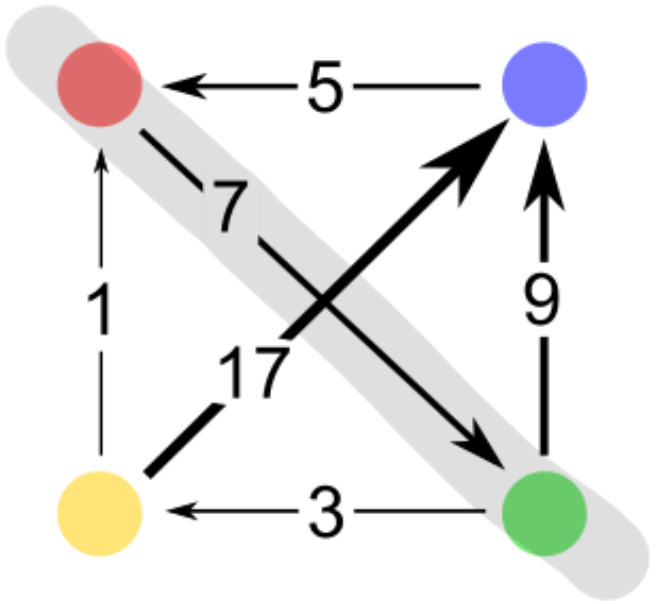

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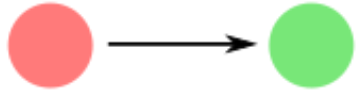

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Schulze

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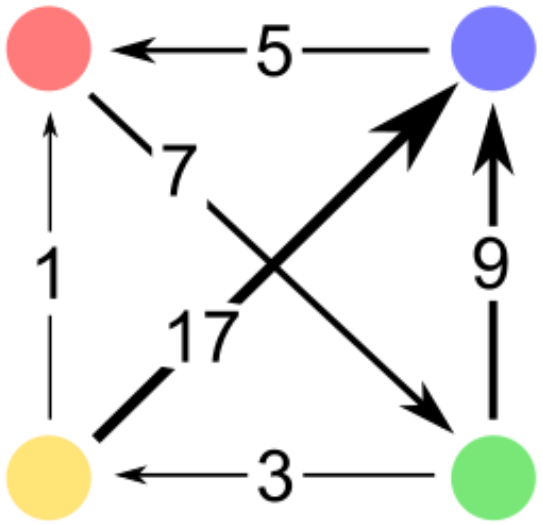

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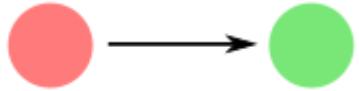

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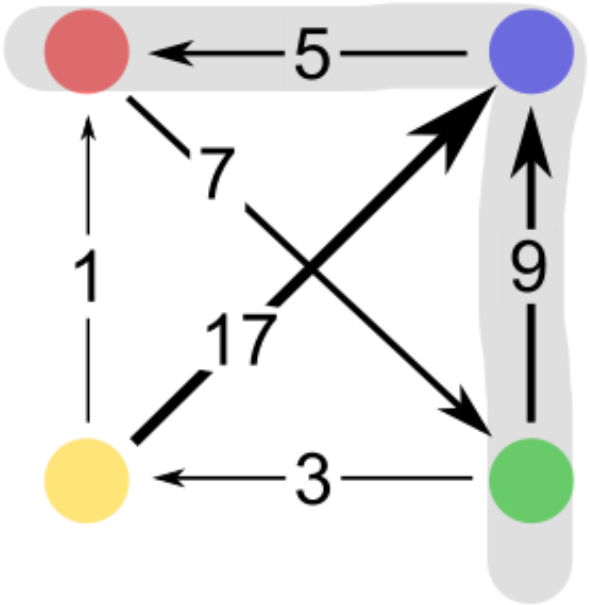

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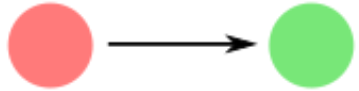

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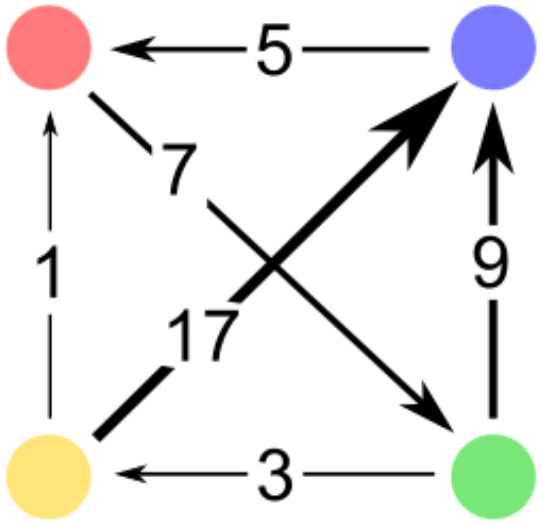

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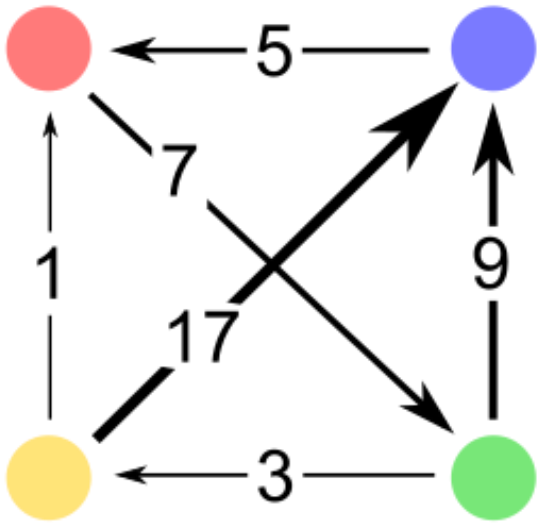

ⓐ>>ⓑ (ⓐ"chain beats"ⓑ) if
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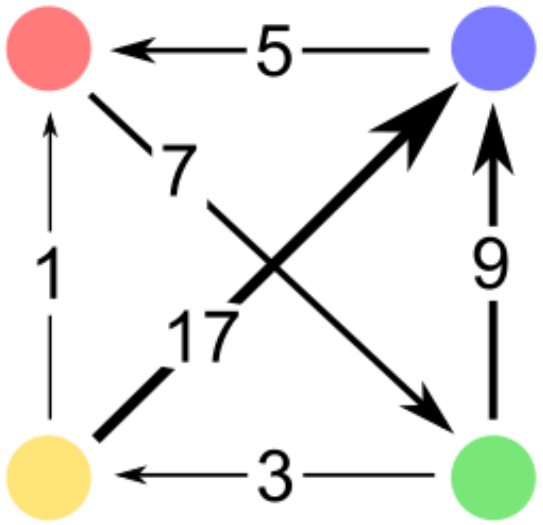

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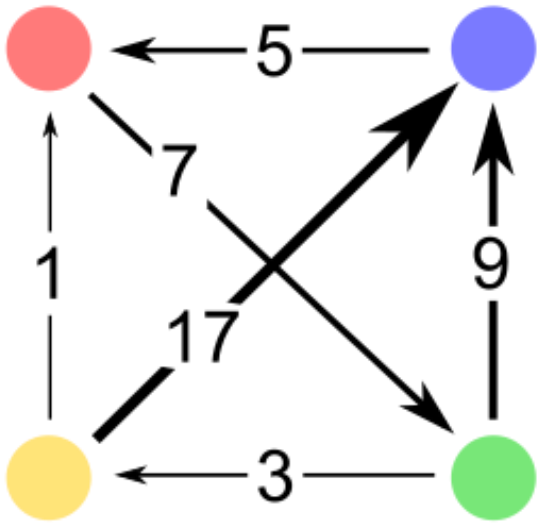

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● >> ●

the strongest path from ● to ● has strength 7

6

Schulze

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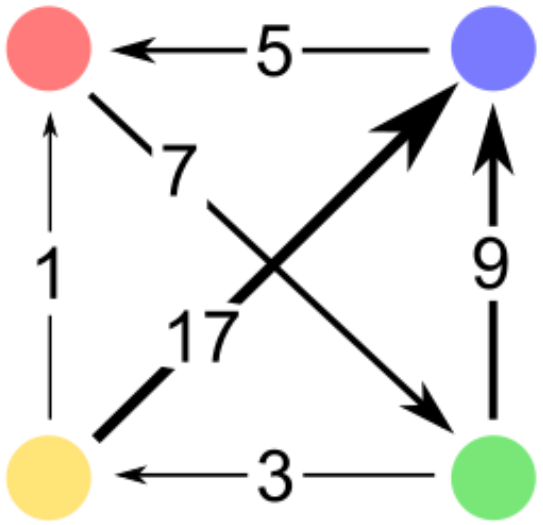

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the strongest path from ● to ● has strength 5

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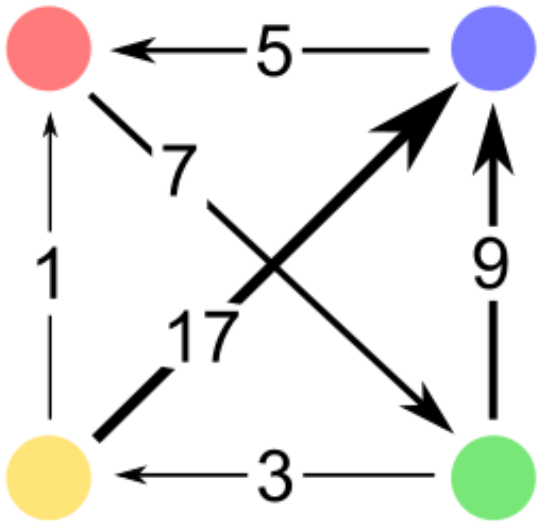

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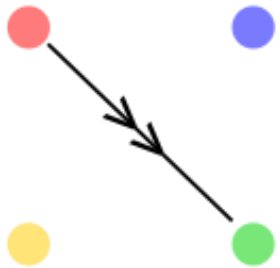

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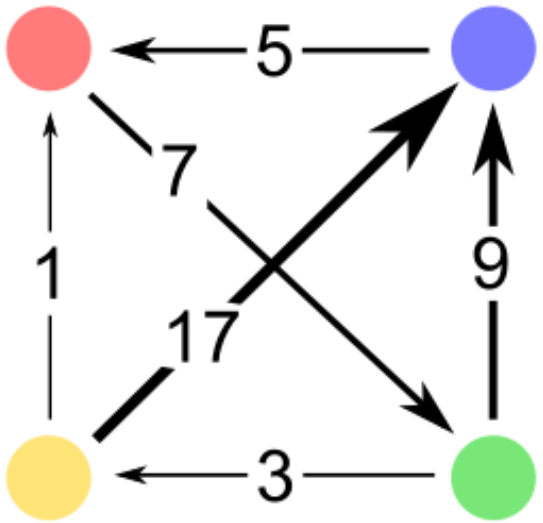

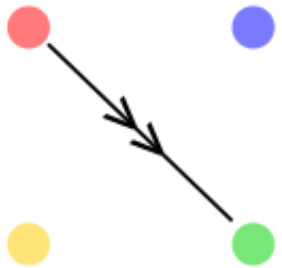

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Schulze

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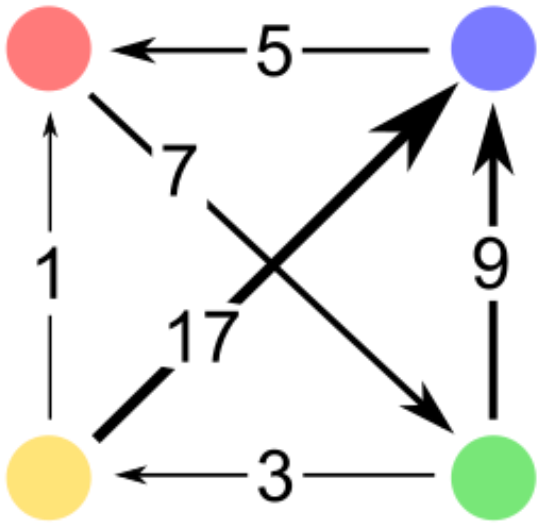

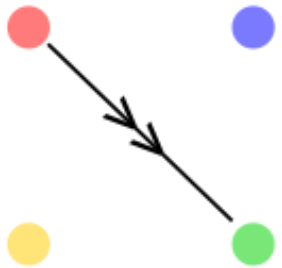

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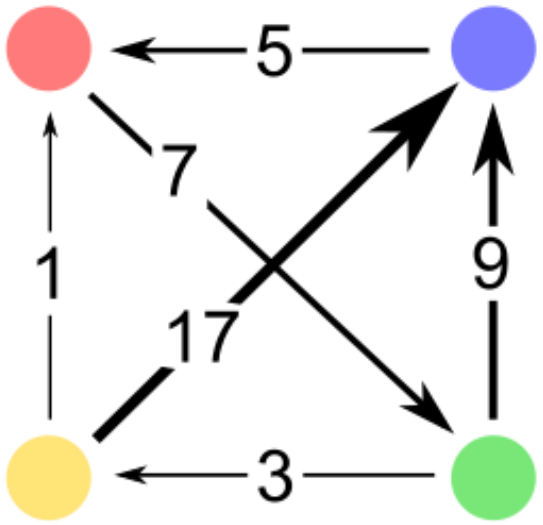

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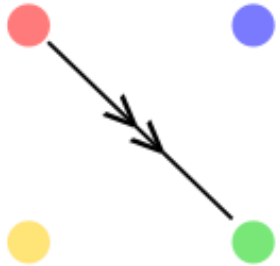

the strongest path from  to  has strength 7

6

Schulze

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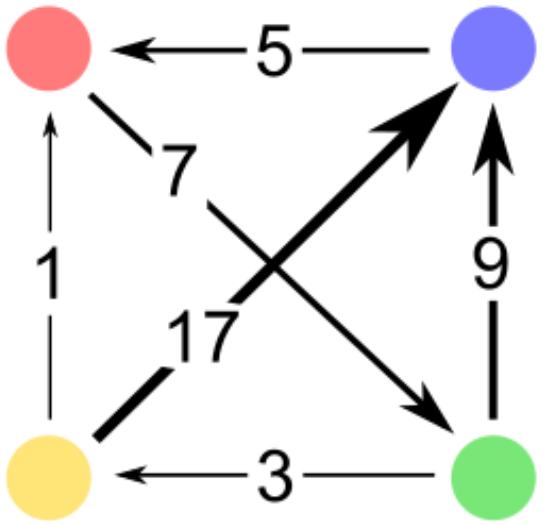

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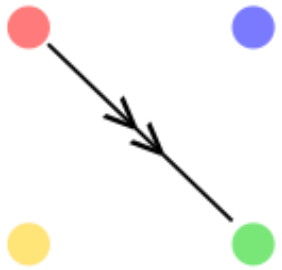

the strongest path from ● to ● has strength 7

the strongest path from ● to ● has strength 5

6

Schulze

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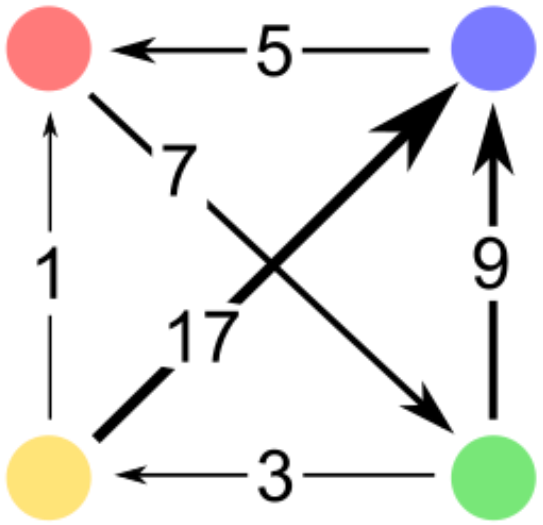

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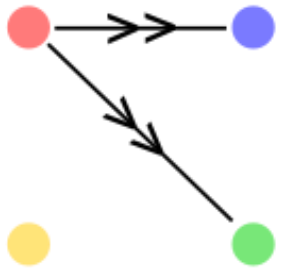

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Schulze

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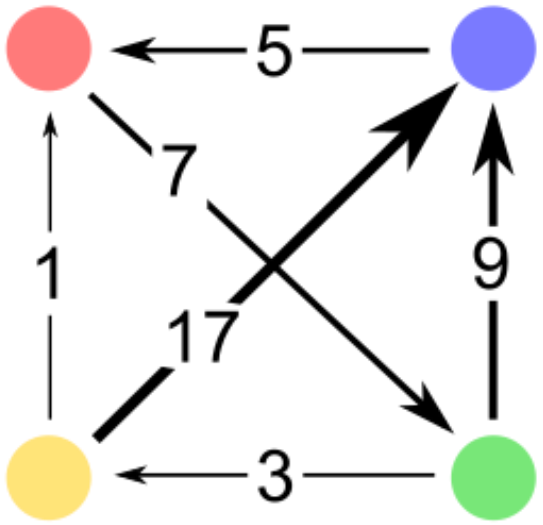

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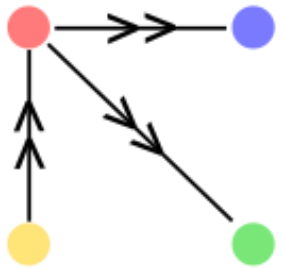

the strongest path from to  has strength 5

the strongest path from  to has strength 3

6

Schulze

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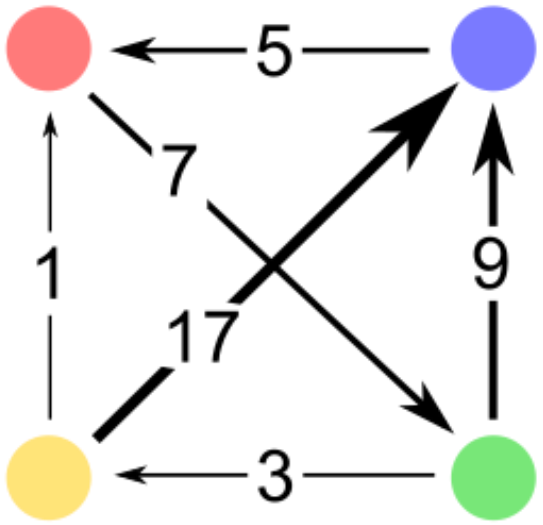

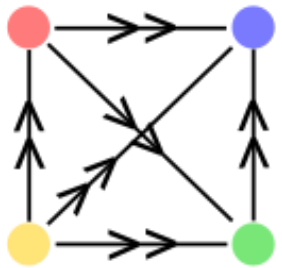

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● >> all others

6

Schulze

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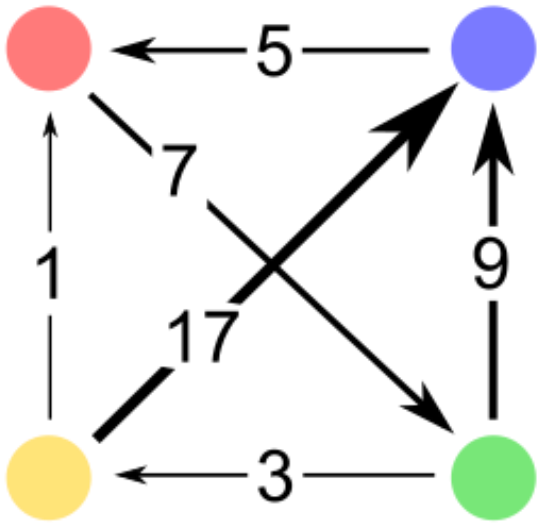

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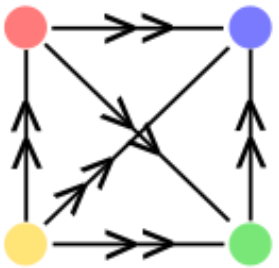

● >> all others

Schulze winner: ●

6

Schulze

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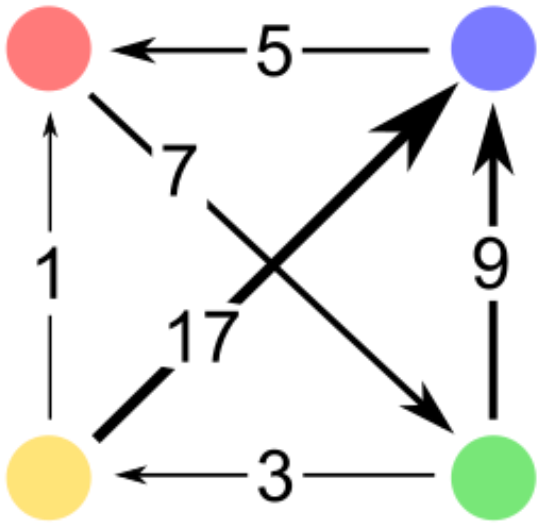

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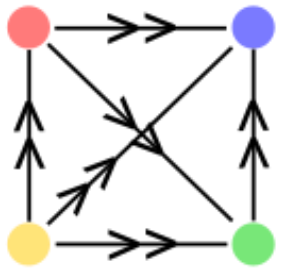
A Schulze winner always exists!



The "chain beats" relation is transitive
Ⓐ>>Ⓑ and Ⓑ>>Ⓒ, then Ⓐ>>Ⓒ

6

Schulze


ubuntu


debian
GNU/Linux

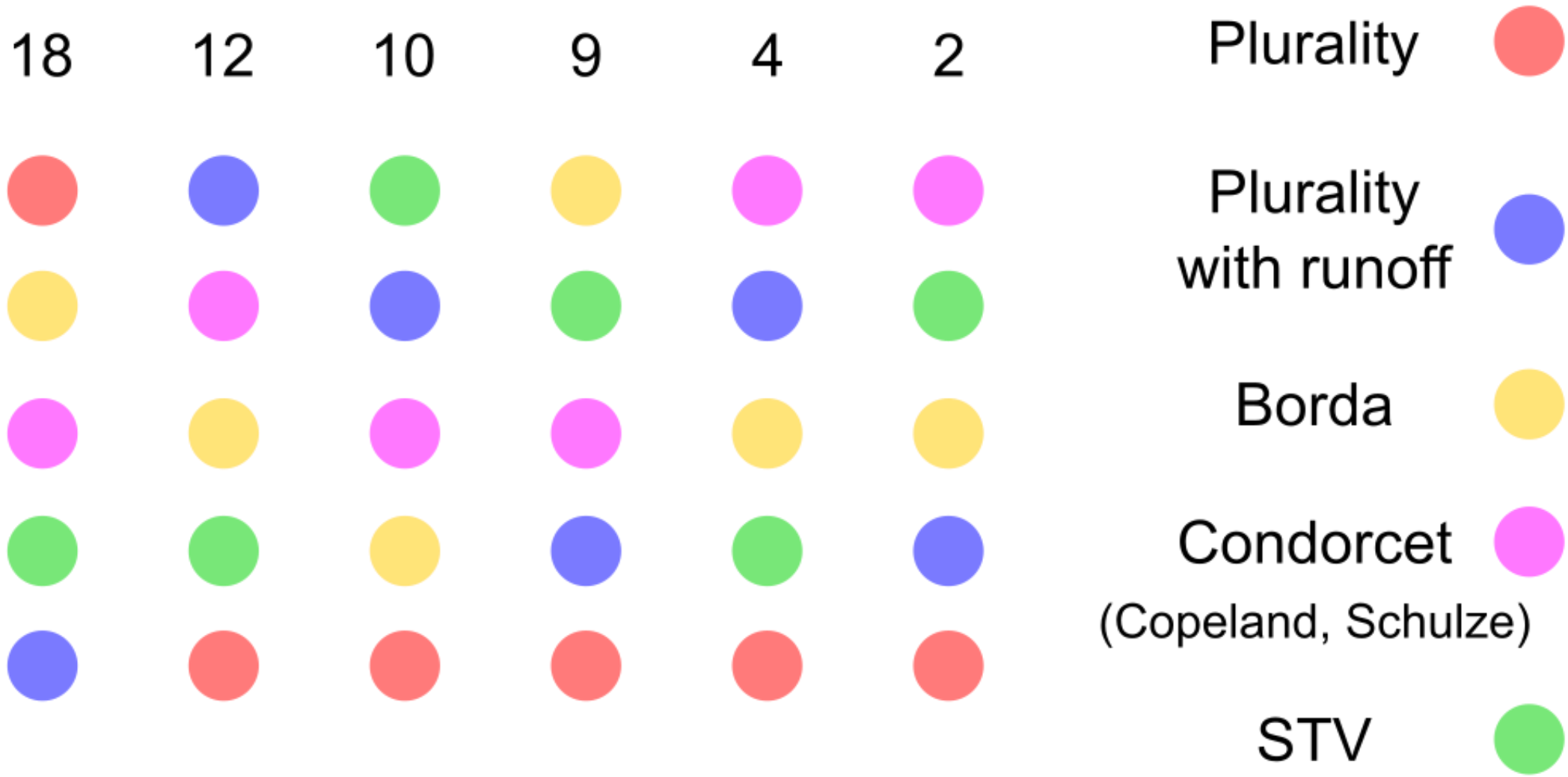

gentoo linux™


IEEE


MTV


acm
Association for
Computing Machinery


topcoder™


18
12
10
9
4
2
Plurality
Plurality
with runoff
Borda
Condorcet
(Copeland, Schulze)
STV

Logistics

Logistics

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Lecture scribing (20%)	Class participation (20%)	Project presentation and report (60%)

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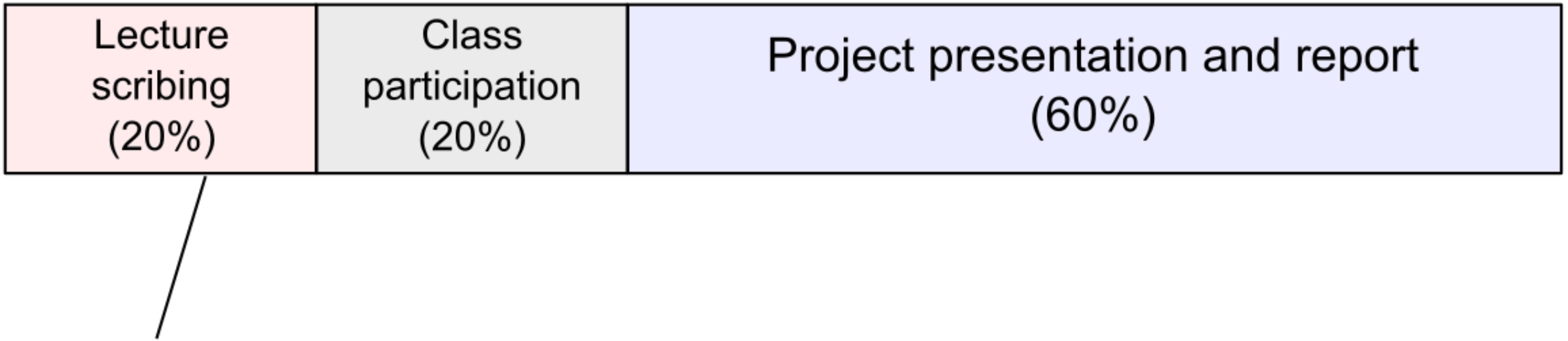

Familiarize yourself with LaTeX!
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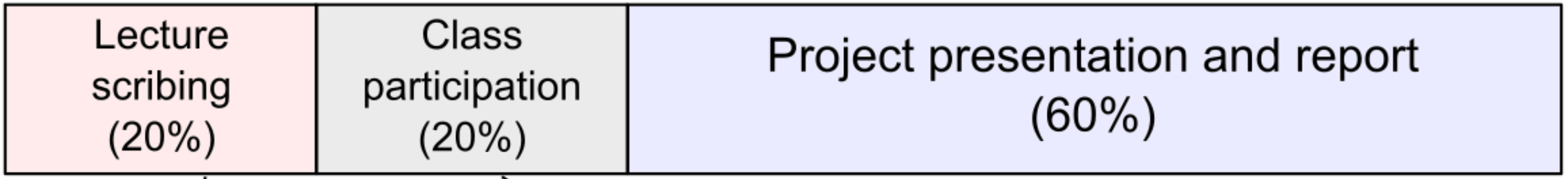

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Discuss among yourselves!

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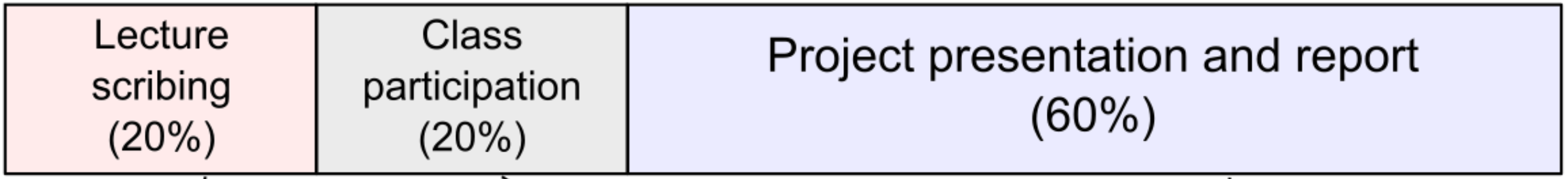

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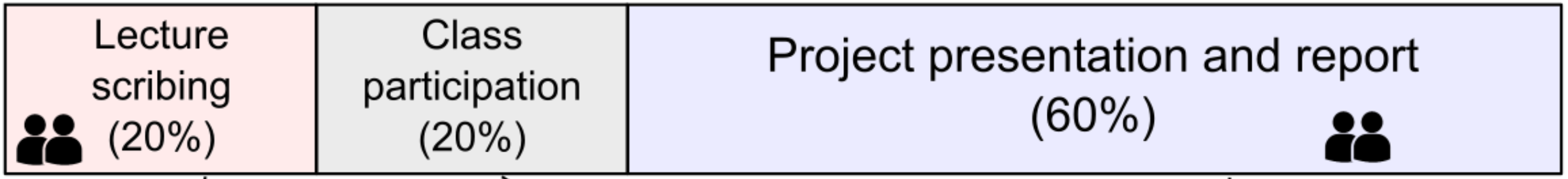

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Next Time

Why truthful voting is impossible

References

- “Disagreement between voting rules” example: http://www.ams.org/publicoutreach/feature-column/fcarc-voting-decision
- The “chain beats” terminology in the description of Schulze rule was borrowed from Hubert Bray’s explanatory video: https://www.youtube.com/watch?v=_HVeN0GnnuA
- The Schulze rule example is from the paper “The Schulze Method of Voting” by Markus Schulze: https://arxiv.org/abs/1804.02973
- (Possibly) strategic voting in the selection of Olympics host: https://www3.nd.edu/~apilking/math10170/Information/Lectures%202015/Topic_2_Plurality_Runoff.pdf